A Market Compliant with COVID-19 Regulations

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Scenario

- We have to automate the checkout desk of a market.
- We have to control when clients enter the checkout area.
- Expected behavior:
  - Clients wait in front of a screen displaying a number or "WAIT".
  - When a number appears, client walks to the corresponding counter.
  - As soon as it passes by the screen, "WAIT" is displayed.
  - When the client reaches the counter, either a new number is displayed (if there are free counters) or "WAIT" (otherwise).
  - When a client leaves, a counter number is displayed.
- Sensors register people movements.
- People behave (no need for physical barriers).
- **Note:** incomplete model. Focus on showing use of sets.

Requirements

REQ 1	The market exit is divided in three areas: the *waiting area*, the *checkout counters* and a *checkout corridor* that connects them.

REQ 2	At most one client can be in the corridor at any time.

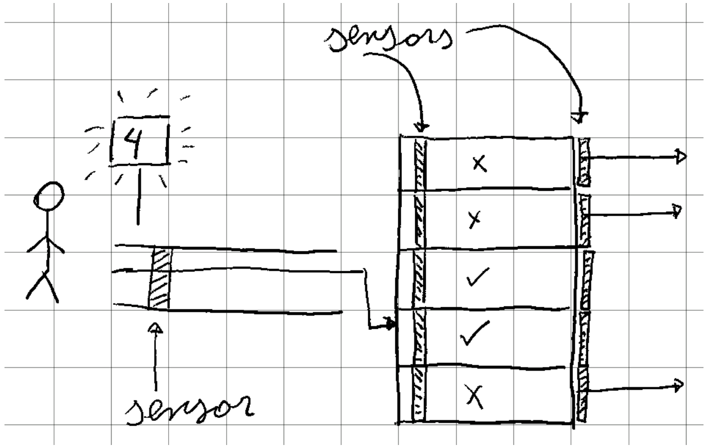
REQ 3	At most one client can be in a checkout counter at any time.

REQ 4	A screen at the entrance of the tells clients to either wait for the corridor to be clear or a counter to be free, or displays the identifier of an available counter.

Requirements

REQ 5	When the corridor is not empty, the screen displays "WAIT".
REQ 6	When no counter is free, the screen displays "WAIT".
REQ 7	When access to the corridor is possible, the screen displays the identifier of one of the available counters.
REQ 8	There are sensors that register people passing at the entrance of the corredor and at the entrance and exit of every counter.

What we see



(Sizes not necessarily proportional)

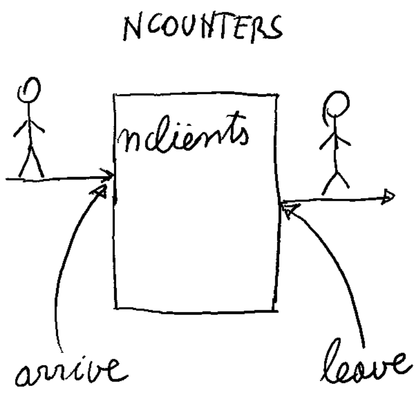
Modeling approach

- As usual: take bird's-eye view.
- Include more requirements, details as we "get closer".
- Do not to overspecify early: refinement may become impossible.

Stages

1. Initial model: just number of clients
2. First refinement: distinguish checkout desks
3. Second refinement: entrance corridor and screen
4. Third refinement: sensors
5. Variant: sets instead of indicator functions

High-level view, visible events



- Clients arrive at the checkout desks.
- Clients leave the checkout desks.
- We only check that we do not have more clients than counters.
- Partial fullfillment of

REQ 9	At most one client can be in a checkout counter at any time.

Model

Context c0

CONSTANTS NCOUNTERS
AXIOMS NCOUNTERS $\in$??

Model

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Machine m0

VARIABLES nclients
INVARIANTS nclients $\in$ 0..NCOUNTERS

Event arrive
  when $nclients < NCOUNTERS$
  then
    $nclients := nclients + 1$
  end

Event leave
  when $nclients > 0$
  then
    $nclients := nclients - 1$
  end

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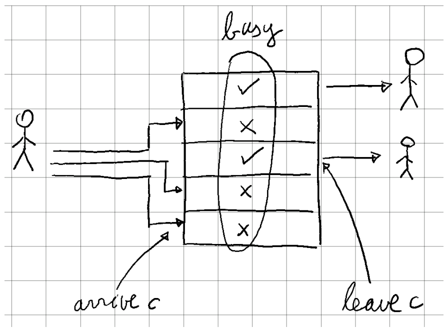
High-level view



- Keep track of (non) available counters.
- Fullfill

REQ 10	At most one client can be in a checkout counter at any time.

- Do not *follow* people.

Model state

- Need to model which counter is available.
- Possibility?

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$$available \in 1..NCOUNTERS \rightarrow \mathrm{BOOL}$$

Model state

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- Possibility?

$$available \in 1..NCOUNTERS \rightarrow \mathrm{BOOL}$$

- But a function $A \rightarrow \mathrm{BOOL}$ denotes a set $S \subseteq A$.
  (it is the *characteristic* or *indicator* function of the set)
- Why not using directly a set?
- The set of busy counters is more useful than the set of available counters (will see later why).
- Do we need it to be $1..NCOUNTERS$?
  - Actually no. We are not going to compare counters.
  - An abstract set will do.

Context c1

EXTENDS c0
SETS COUNTERS
AXIOMS $card(COUNTERS) = NCOUNTERS$

Create it!

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- WD PO not discharged!

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- *card* requires the set to be finite.

  AXIOMS
  $finite(COUNTERS)$
  $card(COUNTERS) = NCOUNTERS$

  (in that order)

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Machine m1

- Refine m0 to track busy counters, create m1.
- SEES c1

VARIABLES busy
INVARIANTS ???

Model state: context and invariants

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$busy \subseteq COUNTERS$

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VARIABLES busy
INVARIANTS
$busy \subseteq COUNTERS$
$card(busy) = nclients$

Events

- Initially, $busy =$

Events

- Initially, $busy = \varnothing$

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- Initially, $busy = \varnothing$
- We see event arrive when some client goes to a free counter and the counter becomes busy.
- An event parameter is the easiest way to model this.

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  refines arrive
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Fill in the Rodin model. POs should become green (otherwise, lasso + P0/ML)

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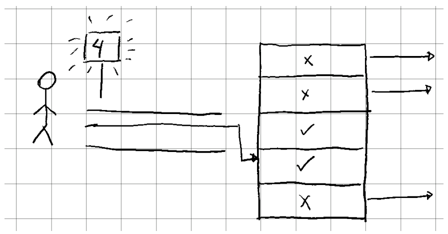
High-level view

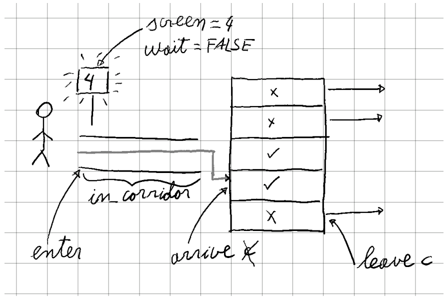


- One-person, one-way corridor: changes contents of screen.
- Selection of available counter via screen.

- Will introduce several components.
- Screen: tells clients what to do (controls entrance to corridor).

Difference with car semaphores: screen goes “red” even if free counters (when people in corridor), then may go “green” again.

Initial model considerations



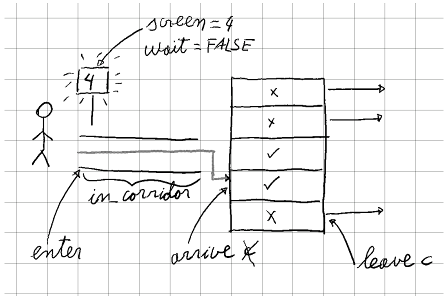
Two variables for display, one for corridor:

- *wait* $\in$ BOOL: clients need to wait?
- *next_counter* $\in$ *COUNTERS*: show free counter / register client destination. (can be used to open physical barrier?).
- *in_corridor* $\in$ BOOL

Relationship below.
Will be captured via invariants.

in_corridor	***wait***	*meaning of* ***next_counter***
FALSE	FALSE	Destination of client (displayed)
FALSE	TRUE	Meaningless (all counters busy, not displayed)
TRUE	FALSE	IMPOSSIBLE
TRUE	TRUE	Destination of client (not displayed)

Initial model considerations



- Introducing event enter.
- Refining events arrive, leave.
- Events & variables model both people, controller.
  - Will be split in next refinement.

Handling the screen

- Could be checked after every state-changing event.
  - Repeated reasoning, models.
  - Specialize events for every situation. (last and non-last car in bridge example)
- Separate events handle screen according to state variables.
- But: additional interleavings, more error possibilities!
- Risky if not verified!

Introducing the model

- Refine m1 into m2.
- New variables and their types:

$$\begin{array}{rcl} \mathit{in\_corridor} & \in & \{0,1\} \\ \mathit{wait} & \in & \mathrm{BOOL} \\ \mathit{next\_counter} & \in & \mathit{COUNTERS} \end{array}$$

- Initialization:

$$\begin{array}{rl} \mathit{in\_corridor} & := \\ \mathit{wait} & := \\ \mathit{next\_counter} & :\in \end{array}$$

Why $\mathit{in\_corridor} \in \{0,1\}$ instead of $\mathit{in\_corridor} \in \mathrm{BOOL}$?

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- Refine m1 into m2.
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Requirements and invariants

REQ 0	When the corridor is not empty, the screen displays "WAIT".

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$$in\_corridor = TRUE \Rightarrow wait = TRUE$$

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REQ 0	When no counter is free, the screen displays "WAIT".

$$busy = COUNTERS \Rightarrow wait = TRUE$$

REQ 0	When access to the corridor is possible, the screen displays the identifier of one of the available counters.

Requirements and invariants

REQ 0	When the corridor is not empty, the screen displays "WAIT".

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REQ 0	When no counter is free, the screen displays "WAIT".

$$busy = COUNTERS \Rightarrow wait = TRUE$$

REQ 0	When access to the corridor is possible, the screen displays the identifier of one of the available counters.

$$wait = FALSE \Rightarrow next\_counter \notin busy$$

Enter them!

The new *enter* and refined *arrive* and *leave*

- leave does not need to be changed.
- A client (can) enters when there is no need to wait.
- The corridor has one more person.
- Other clients have to wait

```
Event enter
  when wait = FALSE
  then
    in_corridor := in_corridor + 1
    wait := TRUE
  end
```

- Arrive at counter previously shown in screen, counter becomes busy.

```
Event arrive
  refines arrive
  when in_corridor > 0
  with c: c = next_counter
  then
    in_corridor := in_corridor − 1
    busy := busy ∪ {next_counter}
  end
```

- Note "c:" in the label of "with": it is necessary (next slide)!
- Type in "enter", modify "arrive

Refining *arrive*

- *next_counter*: see next slide.

```
Event arrive (abstract)
  refines arrive
  any c
  where
    c ∈ COUNTERS
    c ∉ busy
  then
    busy := busy ∪ {c}
  end
```

- Parameter c disappeared: need to state concrete value for it.
- GRD needs to relate guards: prove $in\_corridor > 0 \Rightarrow next\_counter \notin busy$
- If it was a gluing invariant, GRD would
- GRD not discharged.

```
Event arrive (concrete)
  refines arrive
  when in_corridor > 0
  with c: c = next_counter
  then
    in_corridor := in_corridor + 1
    busy := busy ∪ {next_counter}
  end
```

be proven.

- It is! Add it and GRD should be proven.
- Not a requirement, but (a) necessary lemma and (b) sensible.

Screen management

- Display is set to "WAIT" when a client enters.
- We only need to decide whether we allow more clients to enter.

```
Event screen_num
  when


    wait = TRUE
  then
    next_counter :∈ COUNTERS \ busy
    wait := FALSE
  end
```

Type them in
All POs should be fine now.

Screen management

- Display is set to "WAIT" when a client enters.
- We only need to decide whether we allow more clients to enter.

```
Event screen_num
  when
    COUNTERS ≠ busy
    in_corridor = 0
    wait = TRUE
  then
    next_counter :∈ COUNTERS \ busy
    wait := FALSE
  end
```

Type them in
All POs should be fine now.

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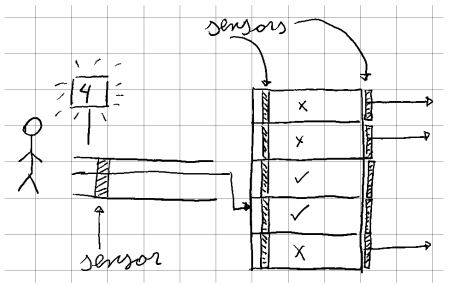
High-level view



- Keep previous "logical" model.
- Add physical model on top, connect with logical model.

- Separate environment and system variables / events.
- Keep interactions clear!
- Guidelines:
  - Some events simulate environment (clients).
  - They react to environment variables and act on sensors.
  - Events that represent the controller.
  - They react to sensors and act on environment variables.

How sensors work

- Not necessarily *real* sensors.
- Client presence activates sensor (a BOOL).
  - Stays on until deactivated by controller.
- Modeling sensor arrays:
  - First idea: use booleans, functions.

$$
\begin{aligned}
S\_E &\in \mathrm{BOOL} \\
S\_A &\in COUNTER \rightarrow \mathrm{BOOL} \\
S\_L &\in COUNTER \rightarrow \mathrm{BOOL}
\end{aligned}
$$

  - *S_E* sensor entry; *S_A* sensor arrival; *S_L* sensor for leaving.
- However, two last ones are indicator sets.
- We can use the **set** of activated sensors.

$$S\_A, S\_L \subseteq COUNTER$$

Using sensors in refined model

- enter, arrive, leave refined.
- **New** events enter_s, arrive_s, leave_s.
  - Note: we will not show leave_s. It is of little interest.
- *_s represent people; they react to environment variables, trigger changes in sensors.
- Modeling agent behavior: variables that represent what people can see, do.

$SCREEN\_CNT \in \{WAIT, NOWAIT\}$	What the screen displays (WAIT or a number)
$CROSSING\_E \in \mathrm{BOOL}$	A person is crossing the corridor sensor
$IN\_CORRIDOR \in \{0, 1\}$	Number of people in the corridor

- *IN_CORRIDOR* could be BOOL. We would then need a gluing invariant with *in_corridor*. Keeping it in $\{0, 1\}$ is easier.

Using sensors in refined model

```
Event enter (abstract)
    refines enter
    when wait = FALSE
    then
        in_corridor := TRUE
        wait := TRUE
    end
```

CROSSING_E in enter_s: a physical person is crossing. Others can see it. We behave correctly.
In enter: controller events should not update environment variables. But we (exceptionally?) model assumption that controllers so fast that when a person has physically crossed, controller has already updated state.

```
Event enter_s
  when SCREEN_CNT = NOWAIT
       CROSSING_E = FALSE
  then
    CROSSING_E := TRUE
    S_E := TRUE
    IN_CORRIDOR := IN_CORRIDOR + 1
  end

Event enter
  refines enter
  when S_E = TRUE // Only look at sensor
  then // abstract actions plus ...
    S_E := FALSE;
    CROSSING_E := FALSE // See explanation
    SCREEN_CNT = WAIT
  end
```

Using sensors in refined model

```
Event arrive (abstract)
  refines arrive
  when in_corridor > 0
  with c: c = next_counter
  then
    in_corridor := FALSE
    busy := busy ∪ {next_counter}
  end
```

CROSSING_E is used here to ensure that a person has actually crossed the entrance and is in the corridor.

```
Event arrive_s
  when IN_CORRIDOR > 0
       CROSSING_E = FALSE // State updated
  then
    IN_CORRIDOR := IN_CORRIDOR − 1
    S_A:= S_A ∪ {next_counter}
  end

Event arrive
  refines arrive
  when next_counter ∈ S_A
  then
    in_corridor := in_corridor − 1
    busy := busy ∪ {next_counter}
    S_A:= S_A \ {next_counter}
end
```

Proof obligations

- Some additional work regarding POs needs to be done.
- IN_CORRIDOR ∈ {0,1} invariant for enter_s.
- GRD for enter, arrive.
- Plus we will introduce a sensible invariant: only one sensor is active at a time:

  inv_sens_arr: ???
- Needs to be discharged for arrive_s

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- GRD for enter, arrive.
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  inv_sens_arr: card(S_A) ≤ 1
- Needs to be discharged for arrive_s

card(S_A) ≤ 1

The (minimal) sequent to discharge (see proving perspective – goal slightly simplified) is

$card(S\_A) \leq 1, IN\_CORRIDOR > 0, CROSSING\_E = FALSE$
$\vdash card(S\_A) \leq card(S\_A \bigcap \{next\_counter\})$

Can be proven if S_A = ∅. Note we have IN_CORRIDOR > 0 and it makes sense that if no one is entering the counter if there is a person in the corridor (see arrive_s). Therefore the invariant

$$IN\_CORRIDOR > 0 \Rightarrow S\_A = \varnothing$$

(if provable) would be helpful. After adding it, proving cardinality is possible with lasso + "remove membership" in the hypothesis IN_CORRIDOR ∈ {0, 1} (click on membership symbol).

$IN\_CORRIDOR > 0 \Rightarrow S\_A = \varnothing$

Invariant needs discharging now in enter_s.
We will delay it.

GRD POs

- GRD POs for enter and arrive are pending.
- They would be

$$next\_counter \in S\_A \Rightarrow in\_corridor > 0$$

for arrive and

$$S\_E = TRUE \Rightarrow wait = FALSE$$

for enter. We will start with the latter.

GRD of enter

- PO for guard strengthening:
  $S\_E = TRUE \Rightarrow wait = FALSE$.
- After positing it as invariant, GRD is proven but the new invariant remains to be proven.
- $SCREEN\_CNT = NOWAIT \Rightarrow wait = FALSE$ as invariant can be proven and helps prove the previous one.

GRD of arrive

- PO for guard strengthening:
  $next\_counter \in S\_A \Rightarrow in\_corridor > 0$.
- Add as invariant. GRD is proven.
- New invariant needs to be discharged for arrive_s.
- Another, intermediate invariant helps prove it:
  $(IN\_CORRIDOR = 1 \wedge CROSSING\_E = FALSE) \Rightarrow in\_corridor = 1$
- At this point, all POs but one should be discharged.

Origin of $(IN\_CORRIDOR = 1 \land CROSSING\_E = FALSE) \Rightarrow in\_corridor = 1$

- The PO in the prover view needs to discharge $S\_A \neq 0 \Rightarrow in\_corridor = 1$.
- Inspecting the hypothesis we have $S\_A \neq 0$. So we need to deduce that $in\_corridor = 1$.
- The rest of the "facts" that we have among the hypotheses are $IN\_CORRIDOR = TRUE$ and $CROSSING\_E = FALSE$.
- Perhaps we can use them to infer $in\_corridor = 1$.

Origin of $(IN\_CORRIDOR = 1 \land CROSSING\_E = FALSE) \Rightarrow in\_corridor = 1$

- Animating the model (see video in web) shows that it is, fundamentally, an event sequence that can fire either leave or screen_num at the end.
- We can make a chart of the state of variables after every event.

	INIT	enter_s	enter	arrive_s	arrive
SCREEN_CNT	NOWAIT	NOWAIT	WAIT	WAIT	WAIT
IN_CORRIDOR	$\bot$	$\top$	$\top$	$\bot$	$\bot$
S_E, CROSSING_E	$\bot$	$\top$	$\bot$	$\bot$	$\bot$
S_A	$\varnothing$	$\varnothing$	$\varnothing$	$\{n\_c\}$	$\varnothing$
in_corridor	0	0	1	1	0
wait	$\bot$	$\bot$	$\top$	$\top$	$\top$
busy	$\varnothing$	$\varnothing$	$\varnothing$	$\varnothing$	$\{n\_c\}$

The two facts we have in our hypotheses ($IN\_CORRIDOR = TRUE$ and $CROSSING\_E = FALSE$) are true only after enter (the state in which arrive_s is executed) and in_corridor = 1. The implication is then a true invariant. Fortunately, it is also an inductive invariant.

Last PO

- Intermediate invariants also helped prove pending POs.
- Pending: $S\_A \neq \varnothing \Rightarrow in\_corridor > 0$.
- Simplifying, it requires proving:

$$next\_counter \in S\_A, \neg S\_A \subseteq \{next\_counter\}, card(S\_A) \leq 1 \vdash in\_corridor > 1$$

- I was not able to discharge it automatically.
- But it should be true – see why.
- Note that $in\_corridor > 1$ cannot be inferred, as $in\_corridor \in \{0, 1\}$.
- Then: prove inconsistency in LHS.
- Since $card(S\_A) \leq 1$, S_A has either one or zero elements.
- Since $next\_counter \in S\_A$, then $S\_A = \{next\_counter\}$
- We have that $\neg) S\_A \subseteq \{next\_counter\})$.
- However, that would mean that $\neg(\{next\_counter\} \subseteq \{next\_counter\})$.
- We have a contradiction and the sequent is proven.
- I have left it as reviewed.
- Model checking (see video) can't find a counterexample, either.