All Rewrite Rules

From Event-B
Jump to navigationJump to search

CAUTION! Any modification to this page shall be announced on the User mailing list!

This page groups together all the rewrite rules implemented (or planned for implementation) in the Rodin prover. The rules themselves can be found in their respective location (for editing purposes):

Conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_rules

Contents

Set Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

	Name	Rule	Side Condition	A/M
*	SIMP_SPECIAL_AND_BTRUE	$P \land \ldots \land \top \land \ldots \land Q \;\widehat{=}\; P \land \ldots \land Q$		A
*	SIMP_SPECIAL_AND_BFALSE	$P \land \ldots \land \bot \land \ldots \land Q \;\widehat{=}\; \bot$		A
*	SIMP_MULTI_AND	$P \land \ldots \land Q \land \ldots \land Q \land \ldots \land R \;\widehat{=}\; P \land \ldots \land Q \land \ldots \land R$		A
*	SIMP_MULTI_AND_NOT	$P \land \ldots \land Q \land \ldots \land \neg Q \land \ldots \land R \;\widehat{=}\; \bot$		A
*	SIMP_SPECIAL_OR_BTRUE	$P \lor \ldots \lor \top \lor \ldots \lor Q \;\widehat{=}\; \top$		A
*	SIMP_SPECIAL_OR_BFALSE	$P \lor \ldots \lor \bot \lor \ldots \lor Q \;\widehat{=}\; P \lor \ldots \lor Q$		A

*	SIMP_MULTI_OR	$P \vee \ldots \vee Q \vee \ldots \vee Q \vee \ldots \vee R \;\widehat{=}\; P \vee \ldots \vee Q \vee \ldots \vee R$		A
*	SIMP_MULTI_OR_NOT	$P \vee \ldots \vee Q \vee \ldots \vee \neg Q \wedge \ldots \wedge R \;\widehat{=}\; \top$		A
*	SIMP_SPECIAL_IMP_BTRUE_R	$P \Rightarrow \top \;\widehat{=}\; \top$		A
*	SIMP_SPECIAL_IMP_BTRUE_L	$\top \Rightarrow P \;\widehat{=}\; P$		A
*	SIMP_SPECIAL_IMP_BFALSE_R	$P \Rightarrow \bot \;\widehat{=}\; \neg P$		A
*	SIMP_SPECIAL_IMP_BFALSE_L	$\bot \Rightarrow P \;\widehat{=}\; \top$		A
*	SIMP_MULTI_IMP	$P \Rightarrow P \;\widehat{=}\; \top$		A
*	SIMP_MULTI_IMP_NOT_L	$\neg P \Rightarrow P \;\widehat{=}\; P$		A
*	SIMP_MULTI_IMP_NOT_R	$P \Rightarrow \neg P \;\widehat{=}\; \neg P$		A
*	SIMP_MULTI_IMP_AND	$P \wedge \ldots \wedge Q \wedge \ldots \wedge R \Rightarrow Q \;\widehat{=}\; \top$		A
*	SIMP_MULTI_IMP_AND_NOT_R	$P \wedge \ldots \wedge Q \wedge \ldots \wedge R \Rightarrow \neg Q \;\widehat{=}\; \neg (P \wedge \ldots \wedge Q \wedge \ldots \wedge R)$		A
*	SIMP_MULTI_IMP_AND_NOT_L	$P \wedge \ldots \wedge \neg Q \wedge \ldots \wedge R \Rightarrow Q \;\widehat{=}\; \neg (P \wedge \ldots \wedge \neg Q \wedge \ldots \wedge R)$		A
*	SIMP_MULTI_EQV	$P \Leftrightarrow P \;\widehat{=}\; \top$		A
*	SIMP_MULTI_EQV_NOT	$P \Leftrightarrow \neg P \;\widehat{=}\; \bot$		A
*	SIMP_SPECIAL_NOT_BTRUE	$\neg \top \;\widehat{=}\; \bot$		A
*	SIMP_SPECIAL_NOT_BFALSE	$\neg \bot \;\widehat{=}\; \top$		A
*	SIMP_NOT_NOT	$\neg \neg P \;\widehat{=}\; P$		AM
*	SIMP_NOTEQUAL	$E \neq F \;\widehat{=}\; \neg E = F$		A
*	SIMP_NOTIN	$E \notin F \;\widehat{=}\; \neg E \in F$		A
*	SIMP_NOTSUBSET	$E \not\subset F \;\widehat{=}\; \neg E \subset F$		A
*	SIMP_NOTSUBSETEQ	$E \not\subseteq F \;\widehat{=}\; \neg E \subseteq F$		A
*	SIMP_NOT_LE	$\neg a \leq b \;\widehat{=}\; a > b$		A
*	SIMP_NOT_GE	$\neg a \geq b \;\widehat{=}\; a < b$		A
*	SIMP_NOT_LT	$\neg a < b \;\widehat{=}\; a \geq b$		A
*	SIMP_NOT_GT	$\neg a > b \;\widehat{=}\; a \leq b$		A
*	SIMP_SPECIAL_NOT_EQUAL_FALSE_R	$\neg (E = \mathrm{FALSE}) \;\widehat{=}\; (E = \mathrm{TRUE})$		A

*	SIMP_SPECIAL_NOT_EQUAL_FALSE_L	$\neg (\mathrm{FALSE} = E) \;\widehat{=}\; (\mathrm{TRUE} = E)$		A
*	SIMP_SPECIAL_NOT_EQUAL_TRUE_R	$\neg (E = \mathrm{TRUE}) \;\widehat{=}\; (E = \mathrm{FALSE})$		A
*	SIMP_SPECIAL_NOT_EQUAL_TRUE_L	$\neg (\mathrm{TRUE} = E) \;\widehat{=}\; (\mathrm{FALSE} = E)$		A
*	SIMP_FORALL_AND	$\forall x \cdot P \wedge Q \;\widehat{=}\; (\forall x \cdot P) \wedge (\forall x \cdot Q)$		A
*	SIMP_EXISTS_OR	$\exists x \cdot P \vee Q \;\widehat{=}\; (\exists x \cdot P) \vee (\exists x \cdot Q)$		A
*	SIMP_EXISTS_IMP	$\exists x \cdot P \Rightarrow Q \;\widehat{=}\; (\forall x \cdot P) \Rightarrow (\exists x \cdot Q)$		A
*	SIMP_FORALL	$\forall \ldots, z, \ldots \cdot P(z) \;\widehat{=}\; \forall z \cdot P(z)$	Quantified identifiers other than z do not occur in P	A
*	SIMP_EXISTS	$\exists \ldots, z, \ldots \cdot P(z) \;\widehat{=}\; \exists z \cdot P(z)$	Quantified identifiers other than z do not occur in P	A
*	SIMP_MULTI_EQUAL	$E = E \;\widehat{=}\; \top$		A
*	SIMP_MULTI_NOTEQUAL	$E \neq E \;\widehat{=}\; \bot$		A
*	SIMP_EQUAL_MAPSTO	$E \mapsto F = G \mapsto H \;\widehat{=}\; E = G \wedge F = H$		A
*	SIMP_EQUAL_SING	$\{E\} = \{F\} \;\widehat{=}\; E = F$		A
*	SIMP_SPECIAL_EQUAL_TRUE	$\mathrm{TRUE} = \mathrm{FALSE} \;\widehat{=}\; \bot$		A
*	SIMP_TYPE_SUBSETEQ	$S \subseteq Ty \;\widehat{=}\; \top$	where Ty is a type expression	A
*	SIMP_SUBSETEQ_SING	$\{E\} \subseteq S \;\widehat{=}\; E \in S$	where E is a single expression	A
*	SIMP_SPECIAL_SUBSETEQ	$\varnothing \subseteq S \;\widehat{=}\; \top$		A
*	SIMP_MULTI_SUBSETEQ	$S \subseteq S \;\widehat{=}\; \top$		A
*	SIMP_SUBSETEQ_BUNION	$S \subseteq A \cup \ldots \cup S \cup \ldots \cup B \;\widehat{=}\; \top$		A
*	SIMP_SUBSETEQ_BINTER	$A \cap \ldots \cap S \cap \ldots \cap B \subseteq S \;\widehat{=}\; \top$		A
*	DERIV_SUBSETEQ_BUNION	$A \cup \ldots \cup B \subseteq S \;\widehat{=}\; A \subseteq S \wedge \ldots \wedge B \subseteq S$		A
*	DERIV_SUBSETEQ_BINTER	$S \subseteq A \cap \ldots \cap B \;\widehat{=}\; S \subseteq A \wedge \ldots \wedge S \subseteq B$		A
*	SIMP_SPECIAL_IN	$E \in \varnothing \;\widehat{=}\; \bot$		A

*	SIMP_MULTI_IN	$B \in \{A, \ldots, B, \ldots, C\} \;\widehat{=}\; \top$		A
*	SIMP_IN_SING	$E \in \{F\} \;\widehat{=}\; E = F$		A
*	SIMP_MULTI_SETENUM	$\{A, \ldots, B, \ldots, B, \ldots, C\} \;\widehat{=}\; \{A, \ldots, B, \ldots, C\}$		A
*	SIMP_SPECIAL_BINTER	$S \cap \ldots \cap \varnothing \cap \ldots \cap T \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_BINTER	$S \cap \ldots \cap Ty \cap \ldots \cap T \;\widehat{=}\; S \cap \ldots \cap T$	where Ty is a type expression	A
*	SIMP_MULTI_BINTER	$S \cap \ldots \cap T \cap \ldots \cap T \cap \ldots \cap U \;\widehat{=}\; S \cap \ldots \cap T \cap \ldots \cap U$		A
*	SIMP_MULTI_EQUAL_BINTER	$S \cap \ldots \cap T \cap \ldots \cap U = T \;\widehat{=}\; T \subseteq S \cap \ldots \cap U$		A
*	SIMP_SPECIAL_BUNION	$S \cup \ldots \cup \varnothing \cup \ldots \cup T \;\widehat{=}\; S \cup \ldots \cup T$		A
*	SIMP_TYPE_BUNION	$S \cup \ldots \cup Ty \cup \ldots \cup T \;\widehat{=}\; Ty$	where Ty is a type expression	A
*	SIMP_MULTI_BUNION	$S \cup \ldots \cup T \cup \ldots \cup T \cup \ldots \cup U \;\widehat{=}\; S \cup \ldots \cup T \cup \ldots \cup U$		A
*	SIMP_MULTI_EQUAL_BUNION	$S \cup \ldots \cup T \cup \ldots \cup U = T \;\widehat{=}\; S \cup \ldots \cup U \subseteq T$		A
*	SIMP_MULTI_SETMINUS	$S \setminus S \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_SETMINUS_R	$S \setminus \varnothing \;\widehat{=}\; S$		A
*	SIMP_SPECIAL_SETMINUS_L	$\varnothing \setminus S \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_SETMINUS	$S \setminus Ty \;\widehat{=}\; \varnothing$	where Ty is a type expression	A
*	SIMP_TYPE_SETMINUS_SETMINUS	$Ty \setminus (Ty \setminus S) \;\widehat{=}\; S$	where Ty is a type expression	A
*	SIMP_KUNION_POW	$\mathrm{union}(\mathbb{P}(S)) \;\widehat{=}\; S$		A
*	SIMP_KUNION_POW1	$\mathrm{union}(\mathbb{P}_1(S)) \;\widehat{=}\; S$		A
*	SIMP_SPECIAL_KUNION	$\mathrm{union}(\{\varnothing\}) \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_QUNION	$\bigcup x \cdot \bot \mid E \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_KINTER	$\mathrm{inter}(\{\varnothing\}) \;\widehat{=}\; \varnothing$		A
*	SIMP_KINTER_POW	$\mathrm{inter}(\mathbb{P}(S)) \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_POW	$\mathbb{P}(\varnothing) \;\widehat{=}\; \{\varnothing\}$		A

*	SIMP_SPECIAL_POW1	$\mathbb{P}_1(\varnothing) \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_CPROD_R	$S \times \varnothing \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_CPROD_L	$\varnothing \times S \;\widehat{=}\; \varnothing$		A
	SIMP_COMPSET_EQUAL	$\{x, y \cdot x = E(y) \land P(y) \mid F(x,y)\} \;\widehat{=}\; \{y \cdot P(y) \mid F(E(y), y)\}$	where x non free in E and non free in P	A
*	SIMP_COMPSET_IN	$\{x \cdot x \in S \mid x\} \;\widehat{=}\; S$	where x non free in S	A
*	SIMP_COMPSET_SUBSETEQ	$\{x \cdot x \subseteq S \mid x\} \;\widehat{=}\; \mathbb{P}(S)$	where x non free in S	A
*	SIMP_SPECIAL_COMPSET_BFALSE	$\{x \cdot \bot \mid x\} \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_COMPSET_BTRUE	$\{x \cdot \top \mid E\} \;\widehat{=}\; Ty$	where the type of E is Ty and E is a maplet combination of locally-bound, pairwise-distinct bound identifiers	A
*	SIMP_SUBSETEQ_COMPSET_L	$\{x \cdot P(x) \mid E(x)\} \subseteq S \;\widehat{=}\; \forall y \cdot P(y) \Rightarrow E(y) \in S$	where y is fresh	A
*	SIMP_IN_COMPSET	$F \in \{x, y, \ldots \cdot P(x,y,\ldots) \mid E(x,y,\ldots)\} \;\widehat{=}\; \exists x, y, \ldots \cdot P(x,y,\ldots) \land E(x,y,\ldots) = F$	where $x, y, \cdots$ are not free in F	A
*	SIMP_IN_COMPSET_ONEPOINT	$E \in \{x \cdot P(x) \mid x\} \;\widehat{=}\; P(E)$	Equivalent to general simplification followed by One Point Rule application with the last conjunct predicate	A
	SIMP_SUBSETEQ_COMPSET_R	$S \subseteq \{x \cdot P(x) \mid x\} \;\widehat{=}\; \forall y \cdot y \in S \Rightarrow P(y)$	where y non free in $S, \{x \cdot P(x) \mid x\}$	M
*	SIMP_SPECIAL_OVERL	$r \lhd \ldots \lhd \varnothing \lhd \ldots \lhd s \;\widehat{=}\; r \lhd \ldots \lhd s$		A
*	SIMP_SPECIAL_KBOOL_BTRUE	$\mathrm{bool}(\top) \;\widehat{=}\; \mathrm{TRUE}$		A
*	SIMP_SPECIAL_KBOOL_BFALSE	$\mathrm{bool}(\bot) \;\widehat{=}\; \mathrm{FALSE}$		A
	DISTRI_SUBSETEQ_BUNION_SING	$S \cup \{F\} \subseteq T \;\widehat{=}\; S \subseteq T \land F \in T$	where F is a single expression	M
*	DEF_FINITE	$\mathrm{finite}(S) \;\widehat{=}\; \exists n, f \cdot f \in 1 \,..\, n \rightarrowtail\!\!\!\!\!\rightarrow S$		M

*	SIMP_SPECIAL_FINITE	$\mathrm{finite}(\varnothing) \;\widehat{=}\; \top$		A
*	SIMP_FINITE_SETENUM	$\mathrm{finite}(\{a, \ldots, b\}) \;\widehat{=}\; \top$		A
*	SIMP_FINITE_BUNION	$\mathrm{finite}(S \cup T) \;\widehat{=}\; \mathrm{finite}(S) \wedge \mathrm{finite}(T)$		A
	SIMP_FINITE_UNION	$\mathrm{finite}(\mathrm{union}(S)) \;\widehat{=}\; \mathrm{finite}(S) \;\wedge\; (\forall x \cdot x \in S \Rightarrow finite(x))$		M
	SIMP_FINITE_QUNION	$\mathrm{finite}(\bigcup x \cdot P \mid E) \;\widehat{=}\; \mathrm{finite}(\{x \cdot P \mid E\}) \;\wedge\; (\forall x \cdot P \Rightarrow finite(E))$		M
*	SIMP_FINITE_POW	$\mathrm{finite}(\mathbb{P}(S)) \;\widehat{=}\; \mathrm{finite}(S)$		A
*	DERIV_FINITE_CPROD	$\mathrm{finite}(S \times T) \;\widehat{=}\; S = \varnothing \vee T = \varnothing \vee (\mathrm{finite}(S) \wedge \mathrm{finite}(T))$		A
*	SIMP_FINITE_CONVERSE	$\mathrm{finite}(r^{-1}) \;\widehat{=}\; \mathrm{finite}(r)$		A
*	SIMP_FINITE_UPTO	$\mathrm{finite}(a \mathinner{..} b) \;\widehat{=}\; \top$		A
*	SIMP_FINITE_ID	$\mathrm{finite}(\mathrm{id}) \;\widehat{=}\; \mathrm{finite}(S)$	where id has type $\mathbb{P}(S \times S)$	A
*	SIMP_FINITE_ID_DOMRES	$\mathrm{finite}(E \lhd \mathrm{id}) \;\widehat{=}\; \mathrm{finite}(E)$		A
*	SIMP_FINITE_PRJ1	$\mathrm{finite}(\mathrm{prj}_1) \;\widehat{=}\; \mathrm{finite}(S \times T)$	where prj_1 has type $\mathbb{P}(S \times T \times S)$	A
*	SIMP_FINITE_PRJ2	$\mathrm{finite}(\mathrm{prj}_2) \;\widehat{=}\; \mathrm{finite}(S \times T)$	where prj_2 has type $\mathbb{P}(S \times T \times T)$	A
*	SIMP_FINITE_PRJ1_DOMRES	$\mathrm{finite}(E \lhd \mathrm{prj}_1) \;\widehat{=}\; \mathrm{finite}(E)$		A
*	SIMP_FINITE_PRJ2_DOMRES	$\mathrm{finite}(E \lhd \mathrm{prj}_2) \;\widehat{=}\; \mathrm{finite}(E)$		A
*	SIMP_FINITE_NATURAL	$\mathrm{finite}(\mathbb{N}) \;\widehat{=}\; \bot$		A
*	SIMP_FINITE_NATURAL1	$\mathrm{finite}(\mathbb{N}_1) \;\widehat{=}\; \bot$		A
*	SIMP_FINITE_INTEGER	$\mathrm{finite}(\mathbb{Z}) \;\widehat{=}\; \bot$		A
*	SIMP_FINITE_BOOL	$\mathrm{finite}(\mathrm{BOOL}) \;\widehat{=}\; \top$		A
*	SIMP_FINITE_LAMBDA	$\mathrm{finite}(\{x \cdot P \mid E \mapsto F\}) \;\widehat{=}\; \mathrm{finite}(\{x \cdot P \mid E\})$	where E is a maplet combination of bound identifiers and expressions that are not bound by the comprehension set (i.e., E is	A

			syntactically injective) and all identifiers bound by the comprehension set that occur in F also occur in E	
*	SIMP_TYPE_IN	$t \in Ty \;\widehat{=}\; \top$	where Ty is a type expression	A
*	SIMP_SPECIAL_EQV_BTRUE	$P \Leftrightarrow \top \;\widehat{=}\; P$		A
*	SIMP_SPECIAL_EQV_BFALSE	$P \Leftrightarrow \bot \;\widehat{=}\; \neg P$		A
*	DEF_SUBSET	$A \subset B \;\widehat{=}\; A \subseteq B \wedge \neg A = B$		A
*	SIMP_SPECIAL_SUBSET_R	$S \subset \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_SPECIAL_SUBSET_L	$\varnothing \subset S \;\widehat{=}\; \neg\, S = \varnothing$		A
*	SIMP_TYPE_SUBSET_L	$S \subset Ty \;\widehat{=}\; S \neq Ty$	where Ty is a type expression	A
*	SIMP_MULTI_SUBSET	$S \subset S \;\widehat{=}\; \bot$		A
*	SIMP_EQUAL_CONSTR	$\mathrm{constr}(a_1, \ldots, a_n) = \mathrm{constr}(b_1, \ldots, b_n) \;\widehat{=}\; a_1 = b_1 \wedge \ldots \wedge a_n = b_n$	where constr is a datatype constructor	A
*	SIMP_EQUAL_CONSTR_DIFF	$\mathrm{constr}_1(\ldots) = \mathrm{constr}_2(\ldots) \;\widehat{=}\; \bot$	where constr_1 and constr_2 are different datatype constructors	A
*	SIMP_DESTR_CONSTR	$\mathrm{destr}(\mathrm{constr}(a_1, \ldots, a_n)) \;\widehat{=}\; a_i$	where destr is the datatype destructor for the i-th argument of datatype constructor constr	A
*	DISTRI_AND_OR	$P \wedge (Q \vee R) \;\widehat{=}\; (P \wedge Q) \vee (P \wedge R)$		M
*	DISTRI_OR_AND	$P \vee (Q \wedge R) \;\widehat{=}\; (P \vee Q) \wedge (P \vee R)$		M
*	DEF_OR	$P \vee Q \vee \ldots \vee R \;\widehat{=}\; \neg P \Rightarrow (Q \vee \ldots \vee R)$		M
*	DERIV_IMP	$P \Rightarrow Q \;\widehat{=}\; \neg Q \Rightarrow \neg P$		M
*	DERIV_IMP_IMP	$P \Rightarrow (Q \Rightarrow R) \;\widehat{=}\; P \wedge Q \Rightarrow R$		M
*	DISTRI_IMP_AND	$P \Rightarrow (Q \wedge R) \;\widehat{=}\; (P \Rightarrow Q) \wedge (P \Rightarrow R)$		M

*	DISTRI_IMP_OR	$(P \lor Q) \Rightarrow R \;\widehat{=}\; (P \Rightarrow R) \land (Q \Rightarrow R)$		M
*	DEF_EQV	$P \Leftrightarrow Q \;\widehat{=}\; (P \Rightarrow Q) \land (Q \Rightarrow P)$		M
*	DISTRI_NOT_AND	$\neg (P \land Q) \;\widehat{=}\; \neg P \lor \neg Q$		M
*	DISTRI_NOT_OR	$\neg (P \lor Q) \;\widehat{=}\; \neg P \land \neg Q$		M
*	DERIV_NOT_IMP	$\neg (P \Rightarrow Q) \;\widehat{=}\; P \land \neg Q$		M
*	DERIV_NOT_FORALL	$\neg \forall x \cdot P \;\widehat{=}\; \exists x \cdot \neg P$		M
*	DERIV_NOT_EXISTS	$\neg \exists x \cdot P \;\widehat{=}\; \forall x \cdot \neg P$		M
*	DEF_IN_MAPSTO	$E \mapsto F \in S \times T \;\widehat{=}\; E \in S \land F \in T$		AM
*	DEF_IN_POW	$E \in \mathbb{P}(S) \;\widehat{=}\; E \subseteq S$		M
*	DEF_IN_POW1	$E \in \mathbb{P}_1(S) \;\widehat{=}\; E \in \mathbb{P}(S) \land S \neq \varnothing$		M
*	DEF_SUBSETEQ	$S \subseteq T \;\widehat{=}\; \forall x \cdot x \in S \Rightarrow x \in T$	where x is not free in S or T	M
*	DEF_IN_BUNION	$E \in S \cup T \;\widehat{=}\; E \in S \lor E \in T$		M
*	DEF_IN_BINTER	$E \in S \cap T \;\widehat{=}\; E \in S \land E \in T$		M
*	DEF_IN_SETMINUS	$E \in S \setminus T \;\widehat{=}\; E \in S \land \neg (E \in T)$		M
*	DEF_IN_SETENUM	$E \in \{A, \ldots, B\} \;\widehat{=}\; E = A \lor \ldots \lor E = B$		M
*	DEF_IN_KUNION	$E \in \mathrm{union}(S) \;\widehat{=}\; \exists s \cdot s \in S \land E \in s$	where s is fresh	M
*	DEF_IN_QUNION	$E \in (\bigcup x \cdot P(x) \mid T(x)) \;\widehat{=}\; \exists s \cdot P(s) \land E \in T(s)$	where s is fresh	M
*	DEF_IN_KINTER	$E \in \mathrm{inter}(S) \;\widehat{=}\; \forall s \cdot s \in S \Rightarrow E \in s$	where s is fresh	M
*	DEF_IN_QINTER	$E \in (\bigcap x \cdot P(x) \mid T(x)) \;\widehat{=}\; \forall s \cdot P(s) \Rightarrow E \in T(s)$	where s is fresh	M
*	DEF_IN_UPTO	$E \in a \mathinner{..} b \;\widehat{=}\; a \leq E \land E \leq b$		M
*	DISTRI_BUNION_BINTER	$S \cup (T \cap U) \;\widehat{=}\; (S \cup T) \cap (S \cup U)$		M
*	DISTRI_BINTER_BUNION	$S \cap (T \cup U) \;\widehat{=}\; (S \cap T) \cup (S \cap U)$		M
	DISTRI_BINTER_SETMINUS	$S \cap (T \setminus U) \;\widehat{=}\; (S \cap T) \setminus (S \cap U)$		M
	DISTRI_SETMINUS_BUNION	$S \setminus (T \cup U) \;\widehat{=}\; S \setminus T \setminus U$		M

*	DERIV_TYPE_SETMINUS_BINTER	$Ty \setminus (S \cap T) \;\widehat{=}\; (Ty \setminus S) \cup (Ty \setminus T)$	where Ty is a type expression	M
*	DERIV_TYPE_SETMINUS_BUNION	$Ty \setminus (S \cup T) \;\widehat{=}\; (Ty \setminus S) \cap (Ty \setminus T)$	where Ty is a type expression	M
*	DERIV_TYPE_SETMINUS_SETMINUS	$Ty \setminus (S \setminus T) \;\widehat{=}\; (Ty \setminus S) \cup T$	where Ty is a type expression	M
	DISTRI_CPROD_BINTER	$S \times (T \cap U) \;\widehat{=}\; (S \times T) \cap (S \times U)$		M
	DISTRI_CPROD_BUNION	$S \times (T \cup U) \;\widehat{=}\; (S \times T) \cup (S \times U)$		M
	DISTRI_CPROD_SETMINUS	$S \times (T \setminus U) \;\widehat{=}\; (S \times T) \setminus (S \times U)$		M
*	DERIV_SUBSETEQ	$S \subseteq T \;\widehat{=}\; (Ty \setminus T) \subseteq (Ty \setminus S)$	where $\mathbb{P}(Ty)$ is the type of S and T	M
*	DERIV_EQUAL	$S = T \;\widehat{=}\; S \subseteq T \land T \subseteq S$	where $\mathbb{P}(Ty)$ is the type of S and T	M
*	DERIV_SUBSETEQ_SETMINUS_L	$A \setminus B \subseteq S \;\widehat{=}\; A \subseteq B \cup S$		M
*	DERIV_SUBSETEQ_SETMINUS_R	$S \subseteq A \setminus B \;\widehat{=}\; S \subseteq A \land S \cap B = \varnothing$		M
*	DEF_PARTITION	$\begin{array}{l}\text{partition}(s, s_1, s_2, \ldots, s_n) \;\widehat{=}\; s = s_1 \cup s_2 \cup \cdots \cup s_n \\ \quad\land\; s_1 \cap s_2 = \varnothing \\ \quad\vdots \\ \quad\land\; s_1 \cap s_n = \varnothing \\ \quad\vdots \\ \quad\land\; s_{n-1} \cap s_n = \varnothing\end{array}$		AM
	SIMP_EMPTY_PARTITION	$\text{partition}(S) \;\widehat{=}\; S = \varnothing$		A
	SIMP_SINGLE_PARTITION	$\text{partition}(S,T) \;\widehat{=}\; S = T$		A

Relation Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

	Name	Rule	Side Condition	A/M
*	SIMP_DOM_SETENUM	$\text{dom}(\{x \mapsto a, \ldots, y \mapsto b\}) \;\widehat{=}\; \{x, \ldots, y\}$		A

*	SIMP_DOM_CONVERSE	$\mathrm{dom}(r^{-1}) \;\widehat{=}\; \mathrm{ran}(r)$		A
*	SIMP_RAN_SETENUM	$\mathrm{ran}(\{x \mapsto a, \ldots, y \mapsto b\}) \;\widehat{=}\; \{a, \ldots, b\}$		A
*	SIMP_RAN_CONVERSE	$\mathrm{ran}(r^{-1}) \;\widehat{=}\; \mathrm{dom}(r)$		A
*	SIMP_SPECIAL_OVERL	$r \lhd\!\!+ \ldots \lhd\!\!+ \varnothing \lhd\!\!+ \ldots \lhd\!\!+ s \;\widehat{=}\; r \lhd\!\!+ \ldots \lhd\!\!+ s$		A
*	SIMP_MULTI_OVERL	$r_1 \lhd\!\!+ \cdots \lhd\!\!+ r_n \widehat{=} r_1 \lhd\!\!+ \cdots \lhd\!\!+ r_{i-1} \lhd\!\!+ r_{i+1} \lhd\!\!+ \cdots \lhd\!\!+ r_n$	there is a j such that $1 \leq i < j \leq n$ and r_i and r_j are syntactically equal.	A
*	SIMP_TYPE_OVERL_CPROD	$r \lhd\!\!+ \cdots \lhd\!\!+ Ty \lhd\!\!+ \cdots \lhd\!\!+ s \;\widehat{=}\; Ty \lhd\!\!+ \cdots \lhd\!\!+ s$	where Ty is a type expression	A
*	SIMP_SPECIAL_DOMRES_L	$\varnothing \lhd r \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_DOMRES_R	$S \lhd \varnothing \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_DOMRES	$Ty \lhd r \;\widehat{=}\; r$	where Ty is a type expression	A
*	SIMP_MULTI_DOMRES_DOM	$\mathrm{dom}(r) \lhd r \;\widehat{=}\; r$		A
*	SIMP_MULTI_DOMRES_RAN	$\mathrm{ran}(r) \lhd r^{-1} \;\widehat{=}\; r^{-1}$		A
*	SIMP_DOMRES_DOMRES_ID	$S \lhd (T \lhd \mathrm{id}) \;\widehat{=}\; (S \cap T) \lhd \mathrm{id}$		A
*	SIMP_DOMRES_DOMSUB_ID	$S \lhd (T \lhd\!\!+ \mathrm{id}) \;\widehat{=}\; (S \setminus T) \lhd \mathrm{id}$		A
*	SIMP_SPECIAL_RANRES_R	$r \rhd \varnothing \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_RANRES_L	$\varnothing \rhd S \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_RANRES	$r \rhd Ty \;\widehat{=}\; r$	where Ty is a type expression	A
*	SIMP_MULTI_RANRES_RAN	$r \rhd \mathrm{ran}(r) \;\widehat{=}\; r$		A
*	SIMP_MULTI_RANRES_DOM	$r^{-1} \rhd \mathrm{dom}(r) \;\widehat{=}\; r^{-1}$		A
*	SIMP_RANRES_ID	$\mathrm{id} \rhd S \;\widehat{=}\; S \lhd \mathrm{id}$		A
*	SIMP_RANSUB_ID	$\mathrm{id} \rhd\!\!+ S \;\widehat{=}\; S \lhd\!\!+ \mathrm{id}$		A
*	SIMP_RANRES_DOMRES_ID	$(S \lhd \mathrm{id}) \rhd T \;\widehat{=}\; (S \cap T) \lhd \mathrm{id}$		A
*	SIMP_RANRES_DOMSUB_ID	$(S \lhd\!\!+ \mathrm{id}) \rhd T \;\widehat{=}\; (T \setminus S) \lhd \mathrm{id}$		A
*	SIMP_SPECIAL_DOMSUB_L	$\varnothing \lhd\!\!+ r \;\widehat{=}\; r$		A
*	SIMP_SPECIAL_DOMSUB_R	$S \lhd\!\!+ \varnothing \;\widehat{=}\; \varnothing$		A

*	SIMP_TYPE_DOMSUB	$Ty \lhd r \;\widehat{=}\; \varnothing$	where Ty is a type expression	A
*	SIMP_MULTI_DOMSUB_DOM	$\mathrm{dom}(r) \lhd r \;\widehat{=}\; \varnothing$		A
*	SIMP_MULTI_DOMSUB_RAN	$\mathrm{ran}(r) \lhd r^{-1} \;\widehat{=}\; \varnothing$		A
*	SIMP_DOMSUB_DOMRES_ID	$S \lhd (T \lhd \mathrm{id}) \;\widehat{=}\; (T \setminus S) \lhd \mathrm{id}$		A
*	SIMP_DOMSUB_DOMSUB_ID	$S \lhd (T \lhd \mathrm{id}) \;\widehat{=}\; (S \cup T) \lhd \mathrm{id}$		A
*	SIMP_SPECIAL_RANSUB_R	$r \rhd \varnothing \;\widehat{=}\; r$		A
*	SIMP_SPECIAL_RANSUB_L	$\varnothing \rhd S \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_RANSUB	$r \rhd Ty \;\widehat{=}\; \varnothing$	where Ty is a type expression	A
*	SIMP_MULTI_RANSUB_DOM	$r^{-1} \rhd \mathrm{dom}(r) \;\widehat{=}\; \varnothing$		A
*	SIMP_MULTI_RANSUB_RAN	$r \rhd \mathrm{ran}(r) \;\widehat{=}\; \varnothing$		A
*	SIMP_RANSUB_DOMRES_ID	$(S \lhd \mathrm{id}) \rhd T \;\widehat{=}\; (S \setminus T) \lhd \mathrm{id}$		A
*	SIMP_RANSUB_DOMSUB_ID	$(S \lhd \mathrm{id}) \rhd T \;\widehat{=}\; (S \cup T) \lhd \mathrm{id}$		A
*	SIMP_SPECIAL_FCOMP	$r \,;\ldots;\, \varnothing \,;\ldots;\, s \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_FCOMP_ID	$r \,;\ldots;\, \mathrm{id} \,;\ldots;\, s \;\widehat{=}\; r \,;\ldots;\, s$		A
*	SIMP_TYPE_FCOMP_R	$r \,;\, Ty \;\widehat{=}\; \mathrm{dom}(r) \times Tb$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_TYPE_FCOMP_L	$Ty \,;\, r \;\widehat{=}\; Ta \times \mathrm{ran}(r)$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_FCOMP_ID	$r \,;\ldots;\, S \lhd \mathrm{id} \,; T \lhd \mathrm{id} \,;\ldots s \;\widehat{=}\; r \,;\ldots;\, (S \cap T) \lhd \mathrm{id} \,;\ldots;\, s$		A
*	SIMP_SPECIAL_BCOMP	$r \circ \ldots \circ \varnothing \circ \ldots \circ s \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_BCOMP_ID	$r \circ \ldots \circ \mathrm{id} \circ \ldots \circ s \;\widehat{=}\; r \circ \ldots \circ s$		A
*	SIMP_TYPE_BCOMP_L	$Ty \circ r \;\widehat{=}\; \mathrm{dom}(r) \times Tb$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_TYPE_BCOMP_R	$r \circ Ty \;\widehat{=}\; Ta \times \mathrm{ran}(r)$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_BCOMP_ID	$r \circ \ldots \circ S \lhd \mathrm{id} \circ T \lhd \mathrm{id} \circ \ldots \circ s \;\widehat{=}\; r \circ \ldots \circ (S \cap T) \lhd \mathrm{id} \circ \ldots \circ s$		A
*	SIMP_SPECIAL_DPROD_R	$r \otimes \varnothing \;\widehat{=}\; \varnothing$		A

*	SIMP_SPECIAL_DPROD_L	$\varnothing \otimes r \;\widehat{=}\; \varnothing$		A
*	SIMP_DPROD_CPROD	$(S \times T) \otimes (U \times V) \;\widehat{=}\; S \cap U \times (T \times V)$		A
*	SIMP_SPECIAL_PPROD_R	$r \parallel \varnothing \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_PPROD_L	$\varnothing \parallel r \;\widehat{=}\; \varnothing$		A
*	SIMP_PPROD_CPROD	$(S \times T) \parallel (U \times V) \;\widehat{=}\; (S \times U) \times (T \times V)$		A
*	SIMP_SPECIAL_RELIMAGE_R	$r[\varnothing] \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_RELIMAGE_L	$\varnothing[S] \;\widehat{=}\; \varnothing$		A
*	SIMP_TYPE_RELIMAGE	$r[Ty] \;\widehat{=}\; \mathrm{ran}(r)$	where Ty is a type expression	A
*	SIMP_MULTI_RELIMAGE_DOM	$r[\mathrm{dom}(r)] \;\widehat{=}\; \mathrm{ran}(r)$		A
*	SIMP_RELIMAGE_ID	$\mathrm{id}[T] \;\widehat{=}\; T$		A
*	SIMP_RELIMAGE_DOMRES_ID	$(S \lhd \mathrm{id})[T] \;\widehat{=}\; S \cap T$		A
*	SIMP_RELIMAGE_DOMSUB_ID	$(S \lhd\!\!\!- \mathrm{id})[T] \;\widehat{=}\; T \setminus S$		A
*	SIMP_MULTI_RELIMAGE_CPROD_SING	$(\{E\} \times S)[\{E\}] \;\widehat{=}\; S$	where E is a single expression	A
*	SIMP_MULTI_RELIMAGE_SING_MAPSTO	$\{E \mapsto F\}[\{E\}] \;\widehat{=}\; \{F\}$	where E is a single expression	A
*	SIMP_MULTI_RELIMAGE_CONVERSE_RANSUB	$(r \rhd\!\!\!- S)^{-1}[S] \;\widehat{=}\; \varnothing$		A
*	SIMP_MULTI_RELIMAGE_CONVERSE_RANRES	$(r \rhd S)^{-1}[S] \;\widehat{=}\; r^{-1}[S]$		A
*	SIMP_RELIMAGE_CONVERSE_DOMSUB	$(S \lhd\!\!\!- r)^{-1}[T] \;\widehat{=}\; r^{-1}[T] \setminus S$		A
	DERIV_RELIMAGE_RANSUB	$(r \rhd\!\!\!- S)[T] \;\widehat{=}\; r[T] \setminus S$		M
	DERIV_RELIMAGE_RANRES	$(r \rhd S)[T] \;\widehat{=}\; r[T] \cap S$		M
*	SIMP_MULTI_RELIMAGE_DOMSUB	$(S \lhd\!\!\!- r)[S] \;\widehat{=}\; \varnothing$		A
	DERIV_RELIMAGE_DOMSUB	$(S \lhd\!\!\!- r)[T] \;\widehat{=}\; r[T \setminus S]$		M
	DERIV_RELIMAGE_DOMRES	$(S \lhd r)[T] \;\widehat{=}\; r[S \cap T]$		M
*	SIMP_SPECIAL_CONVERSE	$\varnothing^{-1} \;\widehat{=}\; \varnothing$		A
*	SIMP_CONVERSE_ID	$\mathrm{id}^{-1} \;\widehat{=}\; \mathrm{id}$		A
*	SIMP_CONVERSE_CPROD	$(S \times T)^{-1} \;\widehat{=}\; T \times S$		A

*	SIMP_CONVERSE_SETENUM	$\{x \mapsto a, \ldots, y \mapsto b\}^{-1} \;\widehat{=}\; \{a \mapsto x, \ldots, b \mapsto y\}$		A
*	SIMP_CONVERSE_COMPSET	$\{X \cdot P \mid x \mapsto y\}^{-1} \;\widehat{=}\; \{X \cdot P \mid y \mapsto x\}$		A
*	SIMP_DOM_ID	$\mathrm{dom}(\mathrm{id}) \;\widehat{=}\; S$	where id has type $\mathbb{P}(S \times S)$	A
*	SIMP_RAN_ID	$\mathrm{ran}(\mathrm{id}) \;\widehat{=}\; S$	where id has type $\mathbb{P}(S \times S)$	A
*	SIMP_FCOMP_ID_L	$(S \lhd \mathrm{id})\,;r \;\widehat{=}\; S \lhd r$		A
*	SIMP_FCOMP_ID_R	$r\,;(S \lhd \mathrm{id}) \;\widehat{=}\; r \rhd S$		A
*	SIMP_SPECIAL_REL_R	$S \leftrightarrow \varnothing \;\widehat{=}\; \{\varnothing\}$	idem for operators $\twoheadleftarrow\!\!\!\!\!\!\rightarrow \;\; \nrightarrow \;\; \rightarrowtail \;\; \twoheadrightarrow\!\!\!\!\!\!\rightarrowtail$	A
*	SIMP_SPECIAL_REL_L	$\varnothing \leftrightarrow S \;\widehat{=}\; \{\varnothing\}$	idem for operators $\twoheadleftarrow\!\!\!\!\!\!\rightarrow \;\; \nrightarrow \;\; \rightarrow \;\; \rightarrowtail \;\; \twoheadrightarrow\!\!\!\!\!\!\rightarrowtail$	A
*	SIMP_FUNIMAGE_PRJ1	$\mathrm{prj}_1(E \mapsto F) \;\widehat{=}\; E$		A
*	SIMP_FUNIMAGE_PRJ2	$\mathrm{prj}_2(E \mapsto F) \;\widehat{=}\; F$		A
*	SIMP_DOM_PRJ1	$\mathrm{dom}(\mathrm{prj}_1) \;\widehat{=}\; S \times T$	where prj_1 has type $\mathbb{P}(S \times T \times S)$	A
*	SIMP_DOM_PRJ2	$\mathrm{dom}(\mathrm{prj}_2) \;\widehat{=}\; S \times T$	where prj_2 has type $\mathbb{P}(S \times T \times T)$	A
*	SIMP_RAN_PRJ1	$\mathrm{ran}(\mathrm{prj}_1) \;\widehat{=}\; S$	where prj_1 has type $\mathbb{P}(S \times T \times S)$	A
*	SIMP_RAN_PRJ2	$\mathrm{ran}(\mathrm{prj}_2) \;\widehat{=}\; T$	where prj_2 has type $\mathbb{P}(S \times T \times T)$	A
*	SIMP_FUNIMAGE_LAMBDA	$(\lambda x \cdot P(x) \mid E(x))(y) \;\widehat{=}\; E(y)$		A
*	SIMP_DOM_LAMBDA	$\mathrm{dom}(\{x \cdot P \mid E \mapsto F) \;\widehat{=}\; \{x \cdot P \mid E\}$		A
*	SIMP_RAN_LAMBDA	$\mathrm{ran}(\{x \cdot P \mid E \mapsto F) \;\widehat{=}\; \{x \cdot P \mid F\}$		A
*	SIMP_IN_FUNIMAGE	$E \mapsto F(E) \in F \;\widehat{=}\; \top$		A
*	SIMP_IN_FUNIMAGE_CONVERSE_L	$F^{-1}(E) \mapsto E \in F \;\widehat{=}\; \top$		A
*	SIMP_IN_FUNIMAGE_CONVERSE_R	$F(E) \mapsto E \in F^{-1} \;\widehat{=}\; \top$		A
*	SIMP_MULTI_FUNIMAGE_SETENUM_LL	$\{A \mapsto E, \ldots, B \mapsto E\}(x) \;\widehat{=}\; E$		A
*	SIMP_MULTI_FUNIMAGE_SETENUM_LR	$\{E, \ldots, x \mapsto y, \ldots, F\}(x) \;\widehat{=}\; y$		A

*	SIMP_MULTI_FUNIMAGE_OVERL_SETENUM	$(r \lhd\!\!\lhd \ldots \lhd\!\!\lhd \{E, \ldots, x \mapsto y, \ldots, F\})(x) \;\widehat{=}\; y$		A
*	SIMP_MULTI_FUNIMAGE_BUNION_SETENUM	$(r \cup \ldots \cup \{E, \ldots, x \mapsto y, \ldots, F\})(x) \;\widehat{=}\; y$		A
*	SIMP_FUNIMAGE_CPROD	$(S \times \{F\})(x) \;\widehat{=}\; F$		A
*	SIMP_FUNIMAGE_ID	$\mathrm{id}(x) \;\widehat{=}\; x$		A
*	SIMP_FUNIMAGE_FUNIMAGE_CONVERSE	$f(f^{-1}(E)) \;\widehat{=}\; E$		A
*	SIMP_FUNIMAGE_CONVERSE_FUNIMAGE	$f^{-1}(f(E)) \;\widehat{=}\; E$		A
*	SIMP_FUNIMAGE_FUNIMAGE_CONVERSE_SETENUM	$\{x \mapsto a, \ldots, y \mapsto b\}(\{a \mapsto x, \ldots, b \mapsto y\}(E)) \;\widehat{=}\; E$		A
*	SIMP_FUNIMAGE_DOMRES	$(E \lhd F)(G) \;\widehat{=}\; F(G)$	with hypothesis $F \in A \;op\; B$ where op is one of $\nrightarrow, \rightarrow, \rightarrowtail, \mapsto, \twoheadrightarrow, \twoheadrightarrow, \rightarrowtail\!\!\!\!\twoheadrightarrow$.	AM
*	SIMP_FUNIMAGE_DOMSUB	$(E \lhd\!\!\!- F)(G) \;\widehat{=}\; F(G)$	with hypothesis $F \in A \;op\; B$ where op is one of $\nrightarrow, \rightarrow, \rightarrowtail, \mapsto, \twoheadrightarrow, \twoheadrightarrow, \rightarrowtail\!\!\!\!\twoheadrightarrow$.	AM
*	SIMP_FUNIMAGE_RANRES	$(F \rhd E)(G) \;\widehat{=}\; F(G)$	with hypothesis $F \in A \;op\; B$ where op is one of $\nrightarrow, \rightarrow, \rightarrowtail, \mapsto, \twoheadrightarrow, \twoheadrightarrow, \rightarrowtail\!\!\!\!\twoheadrightarrow$.	AM
*	SIMP_FUNIMAGE_RANSUB	$(F \rhd\!\!\!- E)(G) \;\widehat{=}\; F(G)$	with hypothesis $F \in A \;op\; B$ where op is one of $\nrightarrow, \rightarrow, \rightarrowtail, \mapsto, \twoheadrightarrow, \twoheadrightarrow, \rightarrowtail\!\!\!\!\twoheadrightarrow$.	AM
*	SIMP_FUNIMAGE_SETMINUS	$(F \setminus E)(G) \;\widehat{=}\; F(G)$	with hypothesis $F \in A \;op\; B$ where op is one of $\nrightarrow, \rightarrow, \rightarrowtail, \mapsto, \twoheadrightarrow, \twoheadrightarrow, \rightarrowtail\!\!\!\!\twoheadrightarrow$.	AM
	DEF_BCOMP	$r \circ \ldots \circ s \;\widehat{=}\; s\,;\ldots;\,r$		M
	DERIV_ID_SING	$\{E\} \lhd \mathrm{id} \;\widehat{=}\; \{E \mapsto E\}$	where E is a single expression	M
*	SIMP_SPECIAL_DOM	$\mathrm{dom}(\varnothing) \;\widehat{=}\; \varnothing$		A
*	SIMP_SPECIAL_RAN	$\mathrm{ran}(\varnothing) \;\widehat{=}\; \varnothing$		A
*	SIMP_CONVERSE_CONVERSE	$r^{-1-1} \;\widehat{=}\; r$		A
*	DERIV_RELIMAGE_FCOMP	$(p\,;q)[s] \;\widehat{=}\; q[p[s]]$		M
*	DERIV_FCOMP_DOMRES	$(s \lhd p)\,;q \;\widehat{=}\; s \lhd (p\,;q)$		M
*	DERIV_FCOMP_DOMSUB	$(s \lhd\!\!\!- p)\,;q \;\widehat{=}\; s \lhd\!\!\!- (p\,;q)$		M

*	DERIV_FCOMP_RANRES	$p ; (q \rhd s) \;\widehat{=}\; (p ; q) \rhd s$		M
*	DERIV_FCOMP_RANSUB	$p ; (q \rhd\!\!> s) \;\widehat{=}\; (p ; q) \rhd\!\!> s$		M
	DERIV_FCOMP_SING	$\{E \mapsto F\} ; \{F \mapsto G\} \;\widehat{=}\; \{E \mapsto G\}$		A
*	SIMP_SPECIAL_EQUAL_RELDOMRAN	$\varnothing \leftrightarrow\!\!\!\!\!\!\rightarrow \varnothing \;\widehat{=}\; \{\varnothing\}$	idem for operators $\rightarrow\!\!\!\!\!\!\rightarrow$	A
*	SIMP_TYPE_DOM	$\mathrm{dom}(Ty) \;\widehat{=}\; Ta$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_TYPE_RAN	$\mathrm{ran}(Ty) \;\widehat{=}\; Tb$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_MULTI_DOM_CPROD	$\mathrm{dom}(E \times E) \;\widehat{=}\; E$		A
*	SIMP_MULTI_RAN_CPROD	$\mathrm{ran}(E \times E) \;\widehat{=}\; E$		A
*	SIMP_MULTI_DOM_DOMRES	$\mathrm{dom}(A \lhd f) \;\widehat{=}\; \mathrm{dom}(f) \cap A$		A
*	SIMP_MULTI_DOM_DOMSUB	$\mathrm{dom}(A \lhd\!\!\!- f) \;\widehat{=}\; \mathrm{dom}(f) \setminus A$		A
*	SIMP_MULTI_RAN_RANRES	$\mathrm{ran}(f \rhd A) \;\widehat{=}\; \mathrm{ran}(f) \cap A$		A
*	SIMP_MULTI_RAN_RANSUB	$\mathrm{ran}(f \rhd\!\!\!- A) \;\widehat{=}\; \mathrm{ran}(f) \setminus A$		A
*	DEF_IN_DOM	$E \in \mathrm{dom}(r) \;\widehat{=}\; \exists y \cdot E \mapsto y \in r$		M
*	DEF_IN_RAN	$F \in \mathrm{ran}(r) \;\widehat{=}\; \exists x \cdot x \mapsto F \in r$		M
*	DEF_IN_CONVERSE	$E \mapsto F \in r^{-1} \;\widehat{=}\; F \mapsto E \in r$		M
*	DEF_IN_DOMRES	$E \mapsto F \in S \lhd r \;\widehat{=}\; E \in S \wedge E \mapsto F \in r$		M
*	DEF_IN_RANRES	$E \mapsto F \in r \rhd T \;\widehat{=}\; E \mapsto F \in r \wedge F \in T$		M
*	DEF_IN_DOMSUB	$E \mapsto F \in S \lhd\!\!\!- r \;\widehat{=}\; E \notin S \wedge E \mapsto F \in r$		M
*	DEF_IN_RANSUB	$E \mapsto F \in r \rhd\!\!\!- T \;\widehat{=}\; E \mapsto F \in r \wedge F \notin T$		M
*	DEF_IN_RELIMAGE	$F \in r[w] \;\widehat{=}\; \exists x \cdot x \in w \wedge x \mapsto F \in r$		M
*	DEF_IN_FCOMP	$E \mapsto F \in (p ; q) \;\widehat{=}\; \exists x \cdot E \mapsto x \in p \wedge x \mapsto F \in q$		M
*	DEF_OVERL	$p \lhd\!\!+ q \;\widehat{=}\; (\mathrm{dom}(q) \lhd\!\!\!- p) \cup q$		M
*	DEF_IN_ID	$E \mapsto F \in \mathrm{id} \;\widehat{=}\; E = F$		M
*	DEF_IN_DPROD	$E \mapsto (F \mapsto G) \in p \otimes q \;\widehat{=}\; E \mapsto F \in p \wedge E \mapsto G \in q$		M

*	DEF_IN_PPROD	$(E \mapsto G) \mapsto (F \mapsto H) \in p \parallel q \;\widehat{=}\; E \mapsto F \in p \wedge G \mapsto H \in q$		M
*	DEF_IN_REL	$r \in S \leftrightarrow T \;\widehat{=}\; r \subseteq S \times T$		M
*	DEF_IN_RELDOM	$r \in S \nleftrightarrow T \;\widehat{=}\; r \in S \leftrightarrow T \wedge \mathrm{dom}(r) = S$		M
*	DEF_IN_RELRAN	$r \in S \leftrightarrow\!\!\!\!\rightarrow T \;\widehat{=}\; r \in S \leftrightarrow T \wedge \mathrm{ran}(r) = T$		M
*	DEF_IN_RELDOMRAN	$r \in S \nleftrightarrow\!\!\!\!\rightarrow T \;\widehat{=}\; r \in S \leftrightarrow T \wedge \mathrm{dom}(r) = S \wedge \mathrm{ran}(r) = T$		M
*	DEF_IN_FCT	$f \in S \nrightarrow T \;\widehat{=}\; f \in S \leftrightarrow T \wedge (\forall x, y, z \cdot x \mapsto y \in f \wedge x \mapsto z \in f \Rightarrow y = z)$		M
*	DEF_IN_TFCT	$f \in S \rightarrow T \;\widehat{=}\; f \in S \nrightarrow T \wedge \mathrm{dom}(f) = S$		M
*	DEF_IN_INJ	$f \in S \rightarrowtail T \;\widehat{=}\; f \in S \nrightarrow T \wedge f^{-1} \in T \nrightarrow S$		M
*	DEF_IN_TINJ	$f \in S \rightarrowtail T \;\widehat{=}\; f \in S \rightarrowtail T \wedge \mathrm{dom}(f) = S$		M
*	DEF_IN_SURJ	$f \in S \twoheadrightarrow T \;\widehat{=}\; f \in S \nrightarrow T \wedge \mathrm{ran}(f) = T$		M
*	DEF_IN_TSURJ	$f \in S \rightarrow\!\!\!\!\!\twoheadrightarrow T \;\widehat{=}\; f \in S \twoheadrightarrow T \wedge \mathrm{dom}(f) = S$		M
*	DEF_IN_BIJ	$f \in S \rightarrowtail\!\!\!\!\!\twoheadrightarrow T \;\widehat{=}\; f \in S \rightarrowtail T \wedge \mathrm{ran}(f) = T$		M
	DISTRI_BCOMP_BUNION	$r \circ (s \cup t) \;\widehat{=}\; (r \circ s) \cup (r \circ t)$		M
*	DISTRI_FCOMP_BUNION_R	$p ; (q \cup r) \;\widehat{=}\; (p ; q) \cup (p ; r)$		M
*	DISTRI_FCOMP_BUNION_L	$(q \cup r) ; p \;\widehat{=}\; (q ; p) \cup (r ; p)$		M
	DISTRI_DPROD_BUNION	$r \otimes (s \cup t) \;\widehat{=}\; (r \otimes s) \cup (r \otimes t)$		M
	DISTRI_DPROD_BINTER	$r \otimes (s \cap t) \;\widehat{=}\; (r \otimes s) \cap (r \otimes t)$		M
	DISTRI_DPROD_SETMINUS	$r \otimes (s \setminus t) \;\widehat{=}\; (r \otimes s) \setminus (r \otimes t)$		M
	DISTRI_DPROD_OVERL	$r \otimes (s \lhd t) \;\widehat{=}\; (r \otimes s) \lhd (r \otimes t)$		M
	DISTRI_PPROD_BUNION	$r \parallel (s \cup t) \;\widehat{=}\; (r \parallel s) \cup (r \parallel t)$		M
	DISTRI_PPROD_BINTER	$r \parallel (s \cap t) \;\widehat{=}\; (r \parallel s) \cap (r \parallel t)$		M
	DISTRI_PPROD_SETMINUS	$r \parallel (s \setminus t) \;\widehat{=}\; (r \parallel s) \setminus (r \parallel t)$		M
	DISTRI_PPROD_OVERL	$r \parallel (s \lhd t) \;\widehat{=}\; (r \parallel s) \lhd (r \parallel t)$		M
	DISTRI_OVERL_BUNION_L	$(p \cup q) \lhd r \;\widehat{=}\; (p \lhd r) \cup (q \lhd r)$		M

	DISTRI_OVERL_BINTER_L	$(p \cap q) \leftleftarrows r \;\widehat{=}\; (p \leftleftarrows r) \cap (q \leftleftarrows r)$		M
*	DISTRI_DOMRES_BUNION_R	$s \lhd (p \cup q) \;\widehat{=}\; (s \lhd p) \cup (s \lhd q)$		M
*	DISTRI_DOMRES_BUNION_L	$(s \cup t) \lhd r \;\widehat{=}\; (s \lhd r) \cup (t \lhd r)$		M
*	DISTRI_DOMRES_BINTER_R	$s \lhd (p \cap q) \;\widehat{=}\; (s \lhd p) \cap (s \lhd q)$		M
*	DISTRI_DOMRES_BINTER_L	$(s \cap t) \lhd r \;\widehat{=}\; (s \lhd r) \cap (t \lhd r)$		M
	DISTRI_DOMRES_SETMINUS_R	$s \lhd (p \setminus q) \;\widehat{=}\; (s \lhd p) \setminus (s \lhd q)$		M
	DISTRI_DOMRES_SETMINUS_L	$(s \setminus t) \lhd r \;\widehat{=}\; (s \lhd r) \setminus (t \lhd r)$		M
	DISTRI_DOMRES_DPROD	$s \lhd (p \otimes q) \;\widehat{=}\; (s \lhd p) \otimes (s \lhd q)$		M
	DISTRI_DOMRES_OVERL	$s \lhd (r \leftleftarrows q) \;\widehat{=}\; (s \lhd r) \leftleftarrows (s \lhd q)$		M
*	DISTRI_DOMSUB_BUNION_R	$s \lhd\!\!\!- (p \cup q) \;\widehat{=}\; (s \lhd\!\!\!- p) \cup (s \lhd\!\!\!- q)$		M
*	DISTRI_DOMSUB_BUNION_L	$(s \cup t) \lhd\!\!\!- r \;\widehat{=}\; (s \lhd\!\!\!- r) \cap (t \lhd\!\!\!- r)$		M
*	DISTRI_DOMSUB_BINTER_R	$s \lhd\!\!\!- (p \cap q) \;\widehat{=}\; (s \lhd\!\!\!- p) \cap (s \lhd\!\!\!- q)$		M
*	DISTRI_DOMSUB_BINTER_L	$(s \cap t) \lhd\!\!\!- r \;\widehat{=}\; (s \lhd\!\!\!- r) \cup (t \lhd\!\!\!- r)$		M
	DISTRI_DOMSUB_DPROD	$A \lhd\!\!\!- (r \otimes s) \;\widehat{=}\; (A \lhd\!\!\!- r) \otimes (A \lhd\!\!\!- s)$		M
	DISTRI_DOMSUB_OVERL	$A \lhd\!\!\!- (r \leftleftarrows s) \;\widehat{=}\; (A \lhd\!\!\!- r) \leftleftarrows (A \lhd\!\!\!- s)$		M
*	DISTRI_RANRES_BUNION_R	$r \rhd (s \cup t) \;\widehat{=}\; (r \rhd s) \cup (r \rhd t)$		M
*	DISTRI_RANRES_BUNION_L	$(p \cup q) \rhd s \;\widehat{=}\; (p \rhd s) \cup (q \rhd s)$		M
*	DISTRI_RANRES_BINTER_R	$r \rhd (s \cap t) \;\widehat{=}\; (r \rhd s) \cap (r \rhd t)$		M
*	DISTRI_RANRES_BINTER_L	$(p \cap q) \rhd s \;\widehat{=}\; (p \rhd s) \cap (q \rhd s)$		M
	DISTRI_RANRES_SETMINUS_R	$r \rhd (s \setminus t) \;\widehat{=}\; (r \rhd s) \setminus (r \rhd t)$		M
	DISTRI_RANRES_SETMINUS_L	$(p \setminus q) \rhd s \;\widehat{=}\; (p \rhd s) \setminus (q \rhd s)$		M
*	DISTRI_RANSUB_BUNION_R	$r \rhd\!\!\!- (s \cup t) \;\widehat{=}\; (r \rhd\!\!\!- s) \cap (r \rhd\!\!\!- t)$		M
*	DISTRI_RANSUB_BUNION_L	$(p \cup q) \rhd\!\!\!- s \;\widehat{=}\; (p \rhd\!\!\!- s) \cup (q \rhd\!\!\!- s)$		M
*	DISTRI_RANSUB_BINTER_R	$r \rhd\!\!\!- (s \cap t) \;\widehat{=}\; (r \rhd\!\!\!- s) \cup (r \rhd\!\!\!- t)$		M
*	DISTRI_RANSUB_BINTER_L	$(p \cap q) \rhd\!\!\!- s \;\widehat{=}\; (p \rhd\!\!\!- s) \cap (q \rhd\!\!\!- s)$		M

*	DISTRI_CONVERSE_BUNION	$(p \cup q)^{-1} \;\widehat{=}\; p^{-1} \cup q^{-1}$		M
	DISTRI_CONVERSE_BINTER	$(p \cap q)^{-1} \;\widehat{=}\; p^{-1} \cap q^{-1}$		M
	DISTRI_CONVERSE_SETMINUS	$(r \setminus s)^{-1} \;\widehat{=}\; r^{-1} \setminus s^{-1}$		M
	DISTRI_CONVERSE_BCOMP	$(r \circ s)^{-1} \;\widehat{=}\; (s^{-1} \circ r^{-1})$		M
	DISTRI_CONVERSE_FCOMP	$(p ; q)^{-1} \;\widehat{=}\; (q^{-1} ; p^{-1})$		M
	DISTRI_CONVERSE_PPROD	$(r \parallel s)^{-1} \;\widehat{=}\; r^{-1} \parallel s^{-1}$		M
	DISTRI_CONVERSE_DOMRES	$(s \lhd r)^{-1} \;\widehat{=}\; r^{-1} \rhd s$		M
	DISTRI_CONVERSE_DOMSUB	$(s \lhd\!\!\!- r)^{-1} \;\widehat{=}\; r^{-1} \rhd\!\!\!- s$		M
	DISTRI_CONVERSE_RANRES	$(r \rhd s)^{-1} \;\widehat{=}\; s \lhd r^{-1}$		M
	DISTRI_CONVERSE_RANSUB	$(r \rhd\!\!\!- s)^{-1} \;\widehat{=}\; s \lhd\!\!\!- r^{-1}$		M
*	DISTRI_DOM_BUNION	$\mathrm{dom}(r \cup s) \;\widehat{=}\; \mathrm{dom}(r) \cup \mathrm{dom}(s)$		M
*	DISTRI_RAN_BUNION	$\mathrm{ran}(r \cup s) \;\widehat{=}\; \mathrm{ran}(r) \cup \mathrm{ran}(s)$		M
*	DISTRI_RELIMAGE_BUNION_R	$r[S \cup T] \;\widehat{=}\; r[S] \cup r[T]$		M
*	DISTRI_RELIMAGE_BUNION_L	$(p \cup q)[S] \;\widehat{=}\; p[S] \cup q[S]$		M
*	DERIV_DOM_TOTALREL	$\mathrm{dom}(r) \;\widehat{=}\; E$	with hypothesis $r \in E \;op\; F$, where op is one of $\leftrightarrow\!\!\!\!\!\rightarrow$, $\leftrightarrow\!\!\!\!\!\!\rightarrow$, $\rightarrow$, $\rightarrowtail$, $\twoheadrightarrow$, $\twoheadrightarrow\!\!\!\!\!\rightarrowtail$	M
	DERIV_RAN_SURJREL	$\mathrm{ran}(r) \;\widehat{=}\; F$	with hypothesis $r \in E \;op\; F$, where op is one of $\leftrightarrow\!\!\!\!\!\twoheadrightarrow$, $\leftrightarrow\!\!\!\!\!\!\twoheadrightarrow$, $\twoheadrightarrow$, $\twoheadrightarrow\!\!\!\!\!\rightarrowtail$, $\twoheadrightarrow\!\!\!\!\!\twoheadrightarrow$	M
*	DERIV_PRJ1_SURJ	$\mathrm{prj}_1 \in Ty_1 \;op\; Ty_2 \;\widehat{=}\; \top$	where Ty_1 and Ty_2 are types and op is one of $\leftrightarrow$, $\leftrightarrow\!\!\!\!\!\rightarrow$, $\leftrightarrow\!\!\!\!\!\!\rightarrow$, $\leftrightarrow\!\!\!\!\!\!\!\rightarrow$, $\nrightarrow$, $\rightarrow$, $\twoheadrightarrow\!\!\!\!\!\!\rightarrow$, $\twoheadrightarrow$	A
*	DERIV_PRJ2_SURJ	$\mathrm{prj}_2 \in Ty_1 \;op\; Ty_2 \;\widehat{=}\; \top$	where Ty_1 and Ty_2 are types and op is one of $\leftrightarrow$, $\leftrightarrow\!\!\!\!\!\rightarrow$, $\leftrightarrow\!\!\!\!\!\!\rightarrow$, $\leftrightarrow\!\!\!\!\!\!\!\rightarrow$, $\nrightarrow$, $\rightarrow$, $\twoheadrightarrow\!\!\!\!\!\!\rightarrow$, $\twoheadrightarrow$	A
*	DERIV_ID_BIJ	$\mathrm{id} \in Ty \;op\; Ty \;\widehat{=}\; \top$	where Ty is a type and op is any arrow	A
*	SIMP_MAPSTO_PRJ1_PRJ2	$\mathrm{prj}_1(E) \mapsto \mathrm{prj}_2(E) \;\widehat{=}\; E$		A

	DERIV_EXPAND_PRJS	$E \;\widehat{=}\; \mathrm{prj}_1(E) \mapsto \mathrm{prj}_2(E)$		M
*	SIMP_DOM_SUCC	$\mathrm{dom}(\mathrm{succ}) \;\widehat{=}\; \mathbb{Z}$		A
*	SIMP_RAN_SUCC	$\mathrm{ran}(\mathrm{succ}) \;\widehat{=}\; \mathbb{Z}$		A
*	DERIV_MULTI_IN_BUNION	$E \in A \cup \cdots \cup \{\cdots, E, \cdots\} \cup \cdots \cup B \;\widehat{=}\; \top$		A
*	DERIV_MULTI_IN_SETMINUS	$E \in S \setminus \{\cdots, E, \cdots\} \;\widehat{=}\; \bot$		A
*	DEF_PRED	$\mathrm{pred} \;\widehat{=}\; \mathrm{succ}^{-1}$		A

Empty Set Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

All rewrite rules that match the pattern $\mathbf{P} = \emptyset$ are also applicable to predicates of the form $\mathbf{P} \subseteq \emptyset$ and $\emptyset = \mathbf{P}$, as these predicates are equivalent. All rewrite rules that match the pattern $\mathbf{P} = Ty$ are also applicable to predicates of the form $Ty \subseteq \mathbf{P}$ and $Ty = \mathbf{P}$, as these predicates are equivalent.

	Name	Rule	Side Condition	A/M
*	DEF_SPECIAL_NOT_EQUAL	$\neg\, S = \varnothing \;\widehat{=}\; \exists x \cdot x \in S$	where x is not free in S	M
*	SIMP_SETENUM_EQUAL_EMPTY	$\{A, \ldots, B\} = \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_SPECIAL_EQUAL_COMPSET	$\{x \cdot P(x) \mid E\} = \varnothing \;\widehat{=}\; \forall x \cdot \neg\, P(x)$		A
*	SIMP_BINTER_EQUAL_TYPE	$A \cap \cdots \cap B = Ty \;\widehat{=}\; A = Ty \wedge \cdots \wedge B = Ty$	where Ty is a type expression	A
*	SIMP_BINTER_SING_EQUAL_EMPTY	$A \cap \cdots \cap \{a\} \cap \cdots \cap B = \varnothing \;\widehat{=}\; \neg\, a \in A \cap \cdots \cap B$		A
*	SIMP_BINTER_SETMINUS_EQUAL_EMPTY	$A \cap \cdots \cap (B \setminus C) \cap \cdots \cap D = \varnothing \;\widehat{=}\; (A \cap \cdots \cap B \cap \cdots \cap D) \setminus C = \varnothing$		A
*	SIMP_BUNION_EQUAL_EMPTY	$A \cup \cdots \cup B = \varnothing \;\widehat{=}\; A = \varnothing \wedge \cdots \wedge B = \varnothing$		A
*	SIMP_SETMINUS_EQUAL_EMPTY	$A \setminus B = \varnothing \;\widehat{=}\; A \subseteq B$		A
*	SIMP_SETMINUS_EQUAL_TYPE	$A \setminus B = Ty \;\widehat{=}\; A = Ty \wedge B = \varnothing$	where Ty is a type expression	A
*	SIMP_POW_EQUAL_EMPTY	$\mathbb{P}(S) = \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_POW1_EQUAL_EMPTY	$\mathbb{P}_1(S) = \varnothing \;\widehat{=}\; S = \varnothing$		A
*	SIMP_KINTER_EQUAL_TYPE	$\mathrm{inter}(S) = Ty \;\widehat{=}\; S = \{Ty\}$	where Ty is a type expression	A

*	SIMP_KUNION_EQUAL_EMPTY	$\mathrm{union}(S) = \varnothing \;\widehat{=}\; S \subseteq \{\varnothing\}$		A
*	SIMP_QINTER_EQUAL_TYPE	$(\bigcap x \cdot P(x) \mid E(x)) = Ty \;\widehat{=}\; \forall x \cdot P(x) \Rightarrow E(x) = Ty$	where Ty is a type expression	A
*	SIMP_QUNION_EQUAL_EMPTY	$(\bigcup x \cdot P(x) \mid E(x)) = \varnothing \;\widehat{=}\; \forall x \cdot P(x) \Rightarrow E(x) = \varnothing$		A
*	SIMP_NATURAL_EQUAL_EMPTY	$\mathbb{N} = \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_NATURAL1_EQUAL_EMPTY	$\mathbb{N}_1 = \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_TYPE_EQUAL_EMPTY	$Ty = \varnothing \;\widehat{=}\; \bot$	where Ty is a type expression	A
*	SIMP_CPROD_EQUAL_EMPTY	$S \times T = \varnothing \;\widehat{=}\; S = \varnothing \lor T = \varnothing$		A
*	SIMP_CPROD_EQUAL_TYPE	$S \times T = Ty \;\widehat{=}\; S = Ta \land T = Tb$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_UPTO_EQUAL_EMPTY	$i \mathinner{..} j = \varnothing \;\widehat{=}\; i > j$		A
*	SIMP_UPTO_EQUAL_INTEGER	$i \mathinner{..} j = \mathbb{Z} \;\widehat{=}\; \bot$		A
*	SIMP_UPTO_EQUAL_NATURAL	$i \mathinner{..} j = \mathbb{N} \;\widehat{=}\; \bot$		A
*	SIMP_UPTO_EQUAL_NATURAL1	$i \mathinner{..} j = \mathbb{N}_1 \;\widehat{=}\; \bot$		A
*	SIMP_SPECIAL_EQUAL_REL	$A \leftrightarrow B = \varnothing \;\widehat{=}\; \bot$	idem for operators $\nrightarrow \rightarrowtail \nrightarrow$	A
	SIMP_TYPE_EQUAL_REL	$A \leftrightarrow B = Ty \;\widehat{=}\; A = Ta \land B = Tb$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_SPECIAL_EQUAL_RELDOM	$A \twoheadleftarrow B = \varnothing \;\widehat{=}\; \neg A = \varnothing \land B = \varnothing$	idem for operator $\rightarrow$	A
	SIMP_TYPE_EQUAL_RELDOMRAN	$A \twoheadleftarrow B = Ty \;\widehat{=}\; \bot$	where Ty is a type expression, idem for operator $\leftrightarrow\!\!\!\!\rightarrow, \twoheadleftarrow\!\!\!\!\twoheadrightarrow, \rightarrow, \rightarrowtail, \nrightarrow, \twoheadrightarrow, \rightarrowtail\!\!\!\!\twoheadrightarrow$	A
*	SIMP_SREL_EQUAL_EMPTY	$A \leftrightarrow\!\!\!\!\twoheadrightarrow B = \varnothing \;\widehat{=}\; A = \varnothing \land \neg B = \varnothing$		A
*	SIMP_STREL_EQUAL_EMPTY	$A \twoheadleftarrow\!\!\!\!\twoheadrightarrow B = \varnothing \;\widehat{=}\; (A = \varnothing \Leftrightarrow \neg B = \varnothing)$		A
*	SIMP_DOM_EQUAL_EMPTY	$\mathrm{dom}(r) = \varnothing \;\widehat{=}\; r = \varnothing$		A
*	SIMP_RAN_EQUAL_EMPTY	$\mathrm{ran}(r) = \varnothing \;\widehat{=}\; r = \varnothing$		A
*	SIMP_FCOMP_EQUAL_EMPTY	$p \mathbin{;} q = \varnothing \;\widehat{=}\; \mathrm{ran}(p) \cap \mathrm{dom}(q) = \varnothing$		A
*	SIMP_BCOMP_EQUAL_EMPTY	$p \circ q = \varnothing \;\widehat{=}\; \mathrm{ran}(q) \cap \mathrm{dom}(p) = \varnothing$		A
*	SIMP_DOMRES_EQUAL_EMPTY	$S \lhd r = \varnothing \;\widehat{=}\; \mathrm{dom}(r) \cap S = \varnothing$		A

*	SIMP_DOMRES_EQUAL_TYPE	$S \lhd r = Ty \;\widehat{=}\; S = Ta \land r = Ty$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_DOMSUB_EQUAL_EMPTY	$S \lhd\!\!\!- r = \varnothing \;\widehat{=}\; \mathrm{dom}(r) \subseteq S$		A
*	SIMP_DOMSUB_EQUAL_TYPE	$S \lhd\!\!\!- r = Ty \;\widehat{=}\; S = \varnothing \land r = Ty$	where Ty is a type expression	A
*	SIMP_RANRES_EQUAL_EMPTY	$r \rhd S = \varnothing \;\widehat{=}\; \mathrm{ran}(r) \cap S = \varnothing$		A
*	SIMP_RANRES_EQUAL_TYPE	$r \rhd S = Ty \;\widehat{=}\; S = Tb \land r = Ty$	where Ty is a type expression equal to $Ta \times Tb$	A
*	SIMP_RANSUB_EQUAL_EMPTY	$r \rhd\!\!\!- S = \varnothing \;\widehat{=}\; \mathrm{ran}(r) \subseteq S$		A
*	SIMP_RANSUB_EQUAL_TYPE	$r \rhd\!\!\!- S = Ty \;\widehat{=}\; S = \varnothing \land r = Ty$	where Ty is a type expression	A
*	SIMP_CONVERSE_EQUAL_EMPTY	$r^{-1} = \varnothing \;\widehat{=}\; r = \varnothing$		A
*	SIMP_CONVERSE_EQUAL_TYPE	$r^{-1} = Ty \;\widehat{=}\; r = Ty^{-1}$	where Ty is a type expression	A
*	SIMP_RELIMAGE_EQUAL_EMPTY	$r[S] = \varnothing \;\widehat{=}\; S \lhd r = \varnothing$		A
*	SIMP_OVERL_EQUAL_EMPTY	$r \lhd\!\!\!- \cdots \lhd\!\!\!- s = \varnothing \;\widehat{=}\; r = \varnothing \land \cdots \land s = \varnothing$		A
*	SIMP_DPROD_EQUAL_EMPTY	$p \otimes q = \varnothing \;\widehat{=}\; \mathrm{dom}(p) \cap \mathrm{dom}(q) = \varnothing$		A
*	SIMP_DPROD_EQUAL_TYPE	$p \otimes q = Ty \;\widehat{=}\; p = Ta \times Tb \land q = Ta \times Tc$	where Ty is a type expression equal to $Ta \times (Tb \times Tc)$	A
*	SIMP_PPROD_EQUAL_EMPTY	$p \parallel q = \varnothing \;\widehat{=}\; p = \varnothing \lor q = \varnothing$		A
*	SIMP_PPROD_EQUAL_TYPE	$p \parallel q = Ty \;\widehat{=}\; p = Ta \times Tc \land q = Tb \times Td$	where Ty is a type expression equal to $(Ta \times Tb) \times (Tc \times Td)$	A
*	SIMP_ID_EQUAL_EMPTY	$\mathrm{id} = \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_PRJ1_EQUAL_EMPTY	$\mathrm{prj}_1 = \varnothing \;\widehat{=}\; \bot$		A
*	SIMP_PRJ2_EQUAL_EMPTY	$\mathrm{prj}_2 = \varnothing \;\widehat{=}\; \bot$		A

Arithmetic Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

	Name	Rule	Side Condition	A/M
*	SIMP_SPECIAL_MOD_0	$0 \bmod E \;\widehat{=}\; 0$		A
*	SIMP_SPECIAL_MOD_1	$E \bmod 1 \;\widehat{=}\; 0$		A
*	SIMP_MIN_SING	$\min(\{E\}) \;\widehat{=}\; E$	where E is a single expression	A
*	SIMP_MAX_SING	$\max(\{E\}) \;\widehat{=}\; E$	where E is a single expression	A
*	SIMP_MIN_NATURAL	$\min(\mathbb{N}) \;\widehat{=}\; 0$		A
*	SIMP_MIN_NATURAL1	$\min(\mathbb{N}_1) \;\widehat{=}\; 1$		A
*	SIMP_MIN_BUNION_SING	$\min(S \cup \ldots \cup \{\min(T)\} \cup \ldots \cup U)$ $\widehat{=}\; \min(S \cup \ldots \cup T \cup \ldots \cup U)$		A
*	SIMP_MAX_BUNION_SING	$\max(S \cup \ldots \cup \{\max(T)\} \cup \ldots \cup U)$ $\widehat{=}\; \max(S \cup \ldots \cup T \cup \ldots \cup U)$		A
*	SIMP_MIN_UPTO	$\min(E \mathinner{..} F) \;\widehat{=}\; E$		A
*	SIMP_MAX_UPTO	$\max(E \mathinner{..} F) \;\widehat{=}\; F$		A
*	SIMP_LIT_MIN	$\min(\{E, \ldots, i, \ldots, j, \ldots, H\}) \;\widehat{=}\; \min(\{E, \ldots, i, \ldots, H\})$	where i and j are literals and $i \leq j$	A
*	SIMP_LIT_MAX	$\max(\{E, \ldots, i, \ldots, j, \ldots, H\}) \;\widehat{=}\; \max(\{E, \ldots, i, \ldots, H\})$	where i and j are literals and $i \geq j$	A
*	SIMP_SPECIAL_CARD	$\mathrm{card}(\varnothing) \;\widehat{=}\; 0$		A
*	SIMP_CARD_SING	$\mathrm{card}(\{E\}) \;\widehat{=}\; 1$	where E is a single expression	A
*	SIMP_SPECIAL_EQUAL_CARD	$\mathrm{card}(S) = 0 \;\widehat{=}\; S = \varnothing$		A
*	SIMP_CARD_POW	$\mathrm{card}(\mathbb{P}(S)) \;\widehat{=}\; 2 \mathbin{\widehat{}} \mathrm{card}(S)$		A
*	SIMP_CARD_BUNION	$\mathrm{card}(S \cup T) \;\widehat{=}\; \mathrm{card}(S) + \mathrm{card}(T) - \mathrm{card}(S \cap T)$		A
	SIMP_CARD_SETMINUS	$\mathrm{card}(S \setminus T) \;\widehat{=}\; \mathrm{card}(S) - \mathrm{card}(T)$	with hypotheses $T \subseteq S$ and either $\mathrm{finite}(S)$ or $\mathrm{finite}(T)$	A
	SIMP_CARD_SETMINUS_SETENUM	$\mathrm{card}(S \setminus \{E_1, \ldots, E_n\}) \;\widehat{=}\; \mathrm{card}(S) - \mathrm{card}(\{E_1, \ldots, E_n\})$	with hypotheses $E_i \in S$ for all $i \in 1 \mathinner{..} n$	A

*	SIMP_CARD_CONVERSE	$\mathrm{card}(r^{-1}) \;\widehat{=}\; \mathrm{card}(r)$		A
*	SIMP_CARD_ID	$\mathrm{card}(\mathrm{id}) \;\widehat{=}\; \mathrm{card}(S)$	where id has type $\mathbb{P}(S \times S)$	A
*	SIMP_CARD_ID_DOMRES	$\mathrm{card}(S \lhd \mathrm{id}) \;\widehat{=}\; \mathrm{card}(S)$		A
*	SIMP_CARD_PRJ1	$\mathrm{card}(\mathrm{prj}_1) \;\widehat{=}\; \mathrm{card}(S \times T)$	where prj_1 has type $\mathbb{P}(S \times T \times S)$	A
*	SIMP_CARD_PRJ2	$\mathrm{card}(\mathrm{prj}_2) \;\widehat{=}\; \mathrm{card}(S \times T)$	where prj_2 has type $\mathbb{P}(S \times T \times T)$	A
*	SIMP_CARD_PRJ1_DOMRES	$\mathrm{card}(E \lhd \mathrm{prj}_1) \;\widehat{=}\; \mathrm{card}(E)$		A
*	SIMP_CARD_PRJ2_DOMRES	$\mathrm{card}(E \lhd \mathrm{prj}_2) \;\widehat{=}\; \mathrm{card}(E)$		A
*	SIMP_CARD_LAMBDA	$\mathrm{card}(\{x \cdot P \mid E \mapsto F\}) \;\widehat{=}\; \mathrm{card}(\{x \cdot P \mid E\})$	where E is a maplet combination of bound identifiers and expressions that are not bound by the comprehension set (i.e., E is syntactically injective) and all identifiers bound by the comprehension set that occur in F also occur in E	A
*	SIMP_LIT_CARD_UPTO	$\mathrm{card}(i \mathinner{..} j) \;\widehat{=}\; j - i + 1$	where i and j are literals and $i \leq j$	A
	SIMP_TYPE_CARD	$\mathrm{card}(\mathit{Tenum}) \;\widehat{=}\; N$	where Tenum is a carrier set containing N elements	A
*	SIMP_LIT_GE_CARD_1	$\mathrm{card}(S) \geq 1 \;\widehat{=}\; \neg\, S = \varnothing$		A
*	SIMP_LIT_LE_CARD_1	$1 \leq \mathrm{card}(S) \;\widehat{=}\; \neg\, S = \varnothing$		A
*	SIMP_LIT_LE_CARD_0	$0 \leq \mathrm{card}(S) \;\widehat{=}\; \top$		A
*	SIMP_LIT_GE_CARD_0	$\mathrm{card}(S) \geq 0 \;\widehat{=}\; \top$		A
*	SIMP_LIT_GT_CARD_0	$\mathrm{card}(S) > 0 \;\widehat{=}\; \neg\, S = \varnothing$		A
*	SIMP_LIT_LT_CARD_0	$0 < \mathrm{card}(S) \;\widehat{=}\; \neg\, S = \varnothing$		A
*	SIMP_LIT_EQUAL_CARD_1			A

		$\mathrm{card}(S) = 1 \;\widehat{=}\; \exists x \cdot S = \{x\}$		
*	SIMP_CARD_NATURAL	$\mathrm{card}(S) \in \mathbb{N} \;\widehat{=}\; \top$		A
*	SIMP_CARD_NATURAL1	$\mathrm{card}(S) \in \mathbb{N}_1 \;\widehat{=}\; \neg\, S = \varnothing$		A
*	SIMP_LIT_IN_NATURAL	$i \in \mathbb{N} \;\widehat{=}\; \top$	where i is a non-negative literal	A
*	SIMP_SPECIAL_IN_NATURAL1	$0 \in \mathbb{N}_1 \;\widehat{=}\; \bot$		A
*	SIMP_LIT_IN_NATURAL1	$i \in \mathbb{N}_1 \;\widehat{=}\; \top$	where i is a positive literal	A
*	SIMP_LIT_UPTO	$i \mathinner{..} j \;\widehat{=}\; \varnothing$	where i and j are literals and $j < i$	A
*	SIMP_LIT_IN_MINUS_NATURAL	$-i \in \mathbb{N} \;\widehat{=}\; \bot$	where i is a positive literal	A
*	SIMP_LIT_IN_MINUS_NATURAL1	$-i \in \mathbb{N}_1 \;\widehat{=}\; \bot$	where i is a non-negative literal	A
*	DEF_IN_NATURAL	$x \in \mathbb{N} \;\widehat{=}\; 0 \le x$		M
*	DEF_IN_NATURAL1	$x \in \mathbb{N}_1 \;\widehat{=}\; 1 \le x$		M
*	SIMP_LIT_EQUAL_KBOOL_TRUE	$\mathrm{bool}(P) = \mathrm{TRUE} \;\widehat{=}\; P$		A
*	SIMP_LIT_EQUAL_KBOOL_FALSE	$\mathrm{bool}(P) = \mathrm{FALSE} \;\widehat{=}\; \neg\, P$		A
	DEF_EQUAL_MIN	$E = \min(S) \;\widehat{=}\; E \in S \wedge (\forall x \cdot x \in S \Rightarrow E \le x)$	where x non free in S, E	M
	DEF_EQUAL_MAX	$E = \max(S) \;\widehat{=}\; E \in S \wedge (\forall x \cdot x \in S \Rightarrow E \ge x)$	where x non free in S, E	M
*	SIMP_SPECIAL_PLUS	$E + \ldots + 0 + \ldots + F \;\widehat{=}\; E + \ldots + F$		A
*	SIMP_SPECIAL_MINUS_R	$E - 0 \;\widehat{=}\; E$		A
*	SIMP_SPECIAL_MINUS_L	$0 - E \;\widehat{=}\; -E$		A
*	SIMP_MINUS_MINUS	$-(-E) \;\widehat{=}\; E$		A
*	SIMP_MINUS_UNMINUS	$E - (-F) \;\widehat{=}\; E + F$	where $(-F)$ is a unary minus expression or a negative literal	M
*	SIMP_MULTI_MINUS	$E - E \;\widehat{=}\; 0$		A
*	SIMP_MULTI_MINUS_PLUS_L	$(A + \ldots + C + \ldots + B) - C \;\widehat{=}\; A + \ldots + B$		M

*	SIMP_MULTI_MINUS_PLUS_R	$C-(A+\ldots+C+\ldots+B) \;\widehat{=}\; -(A+\ldots+B)$		M
*	SIMP_MULTI_MINUS_PLUS_PLUS	$(A+\ldots+E+\ldots+B)-(C+\ldots+E+\ldots+D) \;\widehat{=}\; (A+\ldots+B)-(C+\ldots+D)$		M
*	SIMP_MULTI_PLUS_MINUS	$(A+\ldots+D+\ldots+(C-D)+\ldots+B) \;\widehat{=}\; A+\ldots+C+\ldots+B$		M
*	SIMP_MULTI_ARITHREL_PLUS_PLUS	$A+\ldots+E+\ldots+B<C+\ldots+E+\ldots+D \;\widehat{=}\; A+\ldots+B<C+\ldots+D$	where the root relation ($<$ here) is one of $\{=,<,\leq,>,\geq\}$	M
*	SIMP_MULTI_ARITHREL_PLUS_R	$C<A+\ldots+C\ldots+B \;\widehat{=}\; 0<A+\ldots+B$	where the root relation ($<$ here) is one of $\{=,<,\leq,>,\geq\}$	M
*	SIMP_MULTI_ARITHREL_PLUS_L	$A+\ldots+C\ldots+B<C \;\widehat{=}\; A+\ldots+B<0$	where the root relation ($<$ here) is one of $\{=,<,\leq,>,\geq\}$	M
*	SIMP_MULTI_ARITHREL_MINUS_MINUS_R	$A-C<B-C \;\widehat{=}\; A<B$	where the root relation ($<$ here) is one of $\{=,<,\leq,>,\geq\}$	M
*	SIMP_MULTI_ARITHREL_MINUS_MINUS_L	$C-A<C-B \;\widehat{=}\; B<A$	where the root relation ($<$ here) is one of $\{=,<,\leq,>,\geq\}$	M
*	SIMP_SPECIAL_PROD_0	$E*\ldots*0*\ldots*F \;\widehat{=}\; 0$		A
*	SIMP_SPECIAL_PROD_1	$E*\ldots*1*\ldots*F \;\widehat{=}\; E*\ldots*F$		A
*	SIMP_SPECIAL_PROD_MINUS_EVEN	$(-E)*\ldots*(-F) \;\widehat{=}\; E*\ldots*F$	if an even number of $-$	A
*	SIMP_SPECIAL_PROD_MINUS_ODD	$(-E)*\ldots*(-F) \;\widehat{=}\; -(E*\ldots*F)$	if an odd number of $-$	A
*	SIMP_LIT_MINUS	$-(i) \;\widehat{=}\; (-i)$	where i is a literal	A
*	SIMP_LIT_EQUAL	$i=j \;\widehat{=}\; \top \; or \; \bot \; (computation)$	where i and j are literals	A
*	SIMP_LIT_LE	$i\leq j \;\widehat{=}\; \top \; or \; \bot \; (computation)$	where i and j are literals	A
*	SIMP_LIT_LT	$i<j \;\widehat{=}\; \top \; or \; \bot \; (computation)$	where i and j are literals	A
*	SIMP_LIT_GE	$i\geq j \;\widehat{=}\; \top \; or \; \bot \; (computation)$	where i and j are literals	A
*	SIMP_LIT_GT	$i>j \;\widehat{=}\; \top \; or \; \bot \; (computation)$	where i and j are literals	A
*	SIMP_DIV_MINUS	$(-E)\div(-F) \;\widehat{=}\; E\div F$		A
*	SIMP_SPECIAL_DIV_1	$E\div 1 \;\widehat{=}\; E$		A

*	SIMP_SPECIAL_DIV_0	$0 \div E \;\widehat{=}\; 0$		A
*	SIMP_SPECIAL_EXPN_1_R	$E^1 \;\widehat{=}\; E$		A
*	SIMP_SPECIAL_EXPN_1_L	$1^E \;\widehat{=}\; 1$		A
*	SIMP_SPECIAL_EXPN_0	$E^0 \;\widehat{=}\; 1$		A
*	SIMP_MULTI_LE	$E \leq E \;\widehat{=}\; \top$		A
*	SIMP_MULTI_LT	$E < E \;\widehat{=}\; \bot$		A
*	SIMP_MULTI_GE	$E \geq E \;\widehat{=}\; \top$		A
*	SIMP_MULTI_GT	$E > E \;\widehat{=}\; \bot$		A
*	SIMP_MULTI_DIV	$E \div E \;\widehat{=}\; 1$		A
*	SIMP_MULTI_DIV_PROD	$(X * \ldots * E * \ldots * Y) \div E \;\widehat{=}\; X * \ldots * Y$		A
*	SIMP_MULTI_MOD	$E \bmod E \;\widehat{=}\; 0$		A
	DISTRI_PROD_PLUS	$a * (b + c) \;\widehat{=}\; (a * b) + (a * c)$		M
	DISTRI_PROD_MINUS	$a * (b - c) \;\widehat{=}\; (a * b) - (a * c)$		M
	DERIV_NOT_EQUAL	$\neg E = F \;\widehat{=}\; E < F \lor E > F$	E and F must be of Integer type	M

Extension Proof Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

Rewrite Rules

	Name	Rule	Side Condition	A/M
*	SIMP_SPECIAL_COND_BTRUE	$\mathrm{COND}(\top, E_1, E_2) \;\widehat{=}\; E_1$		A
*	SIMP_SPECIAL_COND_BFALSE	$\mathrm{COND}(\bot, E_1, E_2) \;\widehat{=}\; E_2$		A
*	SIMP_MULTI_COND	$\mathrm{COND}(C, E, E) \;\widehat{=}\; E$		A

Inference Rules

	Name	Rule	Side Condition	A/M

*	DATATYPE_DISTINCT_CASE	$\frac{\mathbf{H}, x = c_1(p_{11}, \ldots, p_{1k}) \vdash \mathbf{G} \quad \ldots \quad \mathbf{H}, x = c_n(p_{n1}, \ldots, p_{nl}) \vdash \mathbf{G}}{\mathbf{H} \vdash \mathbf{G}}$	where x has a datatype DT as type and appears free in $\mathbf{G}$, DT has constructors $c_1, \ldots, c_n$, parameters p_{ij} are introduced as fresh identifiers	M
*	DATATYPE_INDUCTION	$\frac{\mathbf{H}, x = c_1(p_1, \ldots, p_k), \mathbf{P}(p_{I_1}), \ldots, \mathbf{P}(p_{I_l}) \vdash \mathbf{P}(x) \quad \ldots}{\mathbf{H} \vdash \mathbf{P}(x)}$	where x has inductive datatype DT as type and appears free in $\mathbf{P}$; $\{p_{I_1}, \ldots, p_{I_l}\} \subseteq \{p_1, \ldots, p_k\}$ are the inductive parameters (if any); an antecedent is created for every constructor c_i of DT; all parameters are introduced as fresh identifiers; examples here	M
