Inference Rules

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CAUTION! Any modification to this page shall be announced on the User mailing list!

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Inference_Rules.

	Name	Rule	Side Condition	A/M
*	HYP	$\frac{}{\mathbf{H},\mathbf{P} \;\vdash\; \mathbf{P}^\dagger}$	see below for $\mathbf{P}^\dagger$	A
*	HYP_OR	$\frac{}{\mathbf{H},\mathbf{Q} \;\vdash\; \mathbf{P} \lor \ldots \lor \mathbf{Q}^\dagger \lor \ldots \lor \mathbf{R}}$	see below for $\mathbf{Q}^\dagger$	A
*	CNTR	$\frac{}{\mathbf{H},\ \mathbf{P},\ \mathbf{nP}^\dagger \;\vdash\; \mathbf{Q}}$	see below for $\mathbf{nP}^\dagger$	A
*	FALSE_HYP	$\frac{}{\mathbf{H},\bot \;\vdash\; \mathbf{P}}$		A
*	TRUE_GOAL	$\frac{}{\mathbf{H} \;\vdash\; \top}$		A
*	FUN_GOAL	$\frac{}{\mathbf{H},\ f \in E\ op\ F \;\vdash\; f \in T_1 \pfun T_2}$	where T_1 and T_2 denote types and op is one of $\pfun$, $\tfun$, $\pinj$, $\tinj$, $\psur$, $\tsur$, $\tbij$.	A
*	FUN_IMAGE_GOAL	$\frac{\mathbf{H},\ f \in S_1\ op\ S_2,\ f(E) \in S_2 \;\vdash\; \mathbf{P}(f(E))}{\mathbf{H},\ f \in S_1\ op\ S_2 \;\vdash\; \mathbf{P}(f(E))}$	where op denotes a set of relations	M

			(any arrow) and $\mathbf{P}$ is WD strict	
	FUN_GOAL_REC	$\dfrac{}{\mathbf{H},\ f \in S_1\ op_1\ (S_2\ op_2\ (\ldots(S_n\ op_n(U\ opf\ V\ ))\ldots))\ \vdash\ \ f(E_1)(E_2)...(E_n) \in T_1 \pfun T_2}$	where T_1 and T_2 denote types, op denotes a set of relations (any arrow) and opf is one of $\pfun$, $\tfun$, $\pinj$, $\tinj$, $\psur$, $\tsur$, $\tbij$.	A
*	DBL_HYP	$\dfrac{\mathbf{H},\ \mathbf{P}\ \vdash\ \mathbf{Q}}{\mathbf{H},\ \mathbf{P},\ \mathbf{P}\ \vdash\ \mathbf{Q}}$		A
*	AND_L	$\dfrac{\mathbf{H}, \mathbf{P}, \mathbf{Q}\ \vdash\ \mathbf{R}}{\mathbf{H},\ \mathbf{P} \land \mathbf{Q}\ \vdash\ \mathbf{R}}$		A
*	AND_R	$\dfrac{\mathbf{H}\ \vdash\ \mathbf{P} \qquad \mathbf{H}\ \vdash\ \mathbf{Q}}{\mathbf{H}\ \vdash\ \mathbf{P}\ \land\ \mathbf{Q}}$		A
	IMP_L1	$\dfrac{\mathbf{H},\ \mathbf{Q},\ \mathbf{P} \land \ldots \land \mathbf{R} \Rightarrow \mathbf{S}\ \vdash\ \mathbf{T}}{\mathbf{H},\ \mathbf{Q},\ \mathbf{P} \land \ldots \land \mathbf{Q} \land \ldots \land \mathbf{R} \Rightarrow \mathbf{S}\ \vdash\ \mathbf{T}}$		A
*	IMP_R	$\dfrac{\mathbf{H}, \mathbf{P}\ \vdash\ \mathbf{Q}}{\mathbf{H}\ \vdash\ \mathbf{P} \Rightarrow \mathbf{Q}}$		A
*	IMP_AND_L	$\dfrac{\mathbf{H}, \mathbf{P} \Rightarrow \mathbf{Q}, \mathbf{P} \Rightarrow \mathbf{R}\ \vdash\ \mathbf{S}}{\mathbf{H},\ \mathbf{P} \Rightarrow \mathbf{Q} \land \mathbf{R}\ \vdash\ \mathbf{S}}$		A
*	IMP_OR_L	$\dfrac{\mathbf{H}, \mathbf{P} \Rightarrow \mathbf{R}, \mathbf{Q} \Rightarrow \mathbf{R}\ \vdash\ \mathbf{S}}{\mathbf{H},\ \mathbf{P} \lor \mathbf{Q} \Rightarrow \mathbf{R}\ \vdash\ \mathbf{S}}$		A
*	AUTO_MH	$\dfrac{\mathbf{H}, \mathbf{P},\ \mathbf{Q} \Rightarrow \mathbf{R}\ \vdash\ \mathbf{S}}{\mathbf{H},\ \mathbf{P},\ \mathbf{P} \land \mathbf{Q} \Rightarrow \mathbf{R}\ \vdash\ \mathbf{S}}$		A

*	NEG_IN_L	$$\frac{\mathbf{H},\ E \in \{a,\ldots,c\},\ \neg\,(E = b)\ \ \vdash\ \ \mathbf{P}}{\mathbf{H},\ E \in \{a,\ldots,b,\ldots,c\}, \neg\,(E = b)\ \ \vdash\ \ \mathbf{P}}$$		A
*	NEG_IN_R	$$\frac{\mathbf{H},\ E \in \{a,\ldots,c\},\ \neg\,(b = E)\ \ \vdash\ \ \mathbf{P}}{\mathbf{H},\ E \in \{a,\ldots,b,\ldots,c\}, \neg\,(b = E)\ \ \vdash\ \ \mathbf{P}}$$		A
*	XST_L	$$\frac{\mathbf{H},\ \mathbf{P(x)}\ \ \vdash\ \ \mathbf{Q}}{\mathbf{H},\ \exists \mathbf{x} \cdot \mathbf{P(x)}\ \ \vdash\ \ \mathbf{Q}}$$		A
*	ALL_R	$$\frac{\mathbf{H}\ \ \vdash\ \ \mathbf{P(x)}}{\mathbf{H}\ \ \vdash\ \ \forall \mathbf{x} \cdot \mathbf{P(x)}}$$		A
*	EQL_LR	$$\frac{\mathbf{H(E)}\ \ \vdash\ \ \mathbf{P(E)}}{\mathbf{H(x)},\ x = E\ \ \vdash\ \ \mathbf{P(x)}}$$	x is a variable which is not free in E	A
*	EQL_RL	$$\frac{\mathbf{H(E)}\ \ \vdash\ \ \mathbf{P(E)}}{\mathbf{H(x)},\ E = x\ \ \vdash\ \ \mathbf{P(x)}}$$	x is a variable which is not free in E	A
	SUBSET_INTER	$$\frac{\mathbf{H},\ \mathbf{T} \subseteq \mathbf{U}\ \ \vdash\ \ \mathbf{G}(\mathbf{S} \cap \ldots \cap \mathbf{T} \cap \ldots \cap \mathbf{V})}{\mathbf{H},\ \mathbf{T} \subseteq \mathbf{U}\ \ \vdash\ \ \mathbf{G}(\mathbf{S} \cap \ldots \cap \mathbf{T} \cap \ldots \cap \mathbf{U} \cap \ldots \cap \mathbf{V})}$$	where $\mathbf{T}$ and $\mathbf{U}$ are not bound by $\mathbf{G}$	A
	IN_INTER	$$\frac{\mathbf{H},\ \mathbf{E} \in \mathbf{T}\ \ \vdash\ \ \mathbf{G}(\mathbf{S} \cap \ldots \cap \{\mathbf{E}\} \cap \ldots \cap \mathbf{U})}{\mathbf{H},\ \mathbf{E} \in \mathbf{T}\ \ \vdash\ \ \mathbf{G}(\mathbf{S} \cap \ldots \cap \{\mathbf{E}\} \cap \ldots \cap \mathbf{T} \cap \ldots \cap \mathbf{U})}$$	where $\mathbf{E}$ and $\mathbf{T}$ are not bound by $\mathbf{G}$	A
	NOTIN_INTER	$$\frac{\mathbf{H},\ \neg\,\mathbf{E} \in \mathbf{T}\ \ \vdash\ \ \mathbf{G}(\varnothing)}{\mathbf{H},\ \neg\,\mathbf{E} \in \mathbf{T}\ \ \vdash\ \ \mathbf{G}(\mathbf{S} \cap \ldots \cap \{\mathbf{E}\} \cap \ldots \cap \mathbf{T} \cap \ldots \cap \mathbf{U})}$$	where $\mathbf{E}$ and $\mathbf{T}$ are not bound by $\mathbf{G}$	A
*	FIN_L_LOWER_BOUND_L	$$\frac{}{\mathbf{H},\ \mathrm{finite}(S)\ \ \vdash\ \ \exists n \cdot (\forall x \cdot x \in S\ \Rightarrow\ n \leq x)}$$	The goal is discharged	A
*	FIN_L_LOWER_BOUND_R	$$\frac{}{\mathbf{H},\ \mathrm{finite}(S)\ \ \vdash\ \ \exists n \cdot (\forall x \cdot x \in S\ \Rightarrow\ x \geq n)}$$	The goal is discharged	A
*	FIN_L_UPPER_BOUND_L		The goal is	A

		$\frac{}{\mathbf{H},\ \mathrm{finite}(S)\ \vdash\ \exists n \cdot (\forall x \cdot x \in S\ \Rightarrow\ n \geq x)}$	discharged	
*	FIN_L_UPPER_BOUND_R	$\frac{}{\mathbf{H},\ \mathrm{finite}(S)\ \vdash\ \exists n \cdot (\forall x \cdot x \in S\ \Rightarrow\ x \leq n)}$	The goal is discharged	A
*	CONTRADICT_L	$\frac{\mathbf{H},\ \neg\,\mathbf{Q}\ \vdash\ \neg\,\mathbf{P}}{\mathbf{H},\ \mathbf{P}\ \vdash\ \mathbf{Q}}$		M
*	CONTRADICT_R	$\frac{\mathbf{H},\ \neg\,\mathbf{Q}\ \vdash\ \bot}{\mathbf{H}\ \vdash\ \mathbf{Q}}$		M
*	CASE	$\frac{\mathbf{H},\ \mathbf{P}\ \vdash\ \mathbf{R}\qquad \ldots \qquad \mathbf{H},\ \mathbf{Q}\ \vdash\ \mathbf{R}}{\mathbf{H},\ \mathbf{P} \lor \ldots \lor \mathbf{Q}\ \vdash\ \mathbf{R}}$		M
*	IMP_CASE	$\frac{\mathbf{H},\ \neg\mathbf{P}\ \vdash\ \mathbf{R}\qquad \mathbf{H},\ \mathbf{Q}\ \vdash\ \mathbf{R}}{\mathbf{H},\ \mathbf{P} \Rightarrow \mathbf{Q}\ \vdash\ \mathbf{R}}$		M
*	MH	$\frac{\mathbf{H}\ \vdash\ \mathbf{P}\qquad \mathbf{H},\ \mathbf{Q}\ \vdash\ \mathbf{R}}{\mathbf{H},\ \mathbf{P} \Rightarrow \mathbf{Q}\ \vdash\ \mathbf{R}}$		M
*	HM	$\frac{\mathbf{H}\ \vdash\ \neg\,\mathbf{Q}\qquad \mathbf{H},\ \neg\,\mathbf{P}\ \vdash\ \mathbf{R}}{\mathbf{H},\ \mathbf{P} \Rightarrow \mathbf{Q}\ \vdash\ \mathbf{R}}$		M
	EQV_LR	$\frac{\mathbf{H(Q)},\ \mathbf{P} \Leftrightarrow \mathbf{Q}\ \vdash\ \mathbf{G(Q)}}{\mathbf{H(P)},\ \mathbf{P} \Leftrightarrow \mathbf{Q}\ \vdash\ \mathbf{G(P)}}$		M
	EQV_RL	$\frac{\mathbf{H(P)},\ \mathbf{P} \Leftrightarrow \mathbf{Q}\ \vdash\ \mathbf{G(P)}}{\mathbf{H(Q)},\ \mathbf{P} \Leftrightarrow \mathbf{Q}\ \vdash\ \mathbf{G(Q)}}$		M
*	OV_SETENUM_L	$\frac{\mathbf{H},\ G = E,\ \mathbf{P}(F)\ \vdash\ \mathbf{Q}\qquad \mathbf{H},\ \neg\,(G = E),\ \mathbf{P}((\{E\}) \lhd f)(G))\ \vdash\ \mathbf{Q}}{\mathbf{H},\ \mathbf{P}((f \Leftarrow \{E \mapsto F\})(G))\ \vdash\ \mathbf{Q}}$	where $\mathbf{P}$ is WD strict	A
*	OV_SETENUM_R	$\frac{\mathbf{H},\ G = E\ \vdash\ \mathbf{Q}(F)\qquad \mathbf{H},\ \neg\,(G = E)\ \vdash\ \mathbf{Q}((\{E\}) \lhd f)(G))}{\mathbf{H}\ \vdash\ \mathbf{Q}((f \Leftarrow \{E \mapsto F\})(G))}$	where $\mathbf{Q}$ is WD strict	A

*	OV_L	$$\frac{\mathbf{H},\ G \in \mathrm{dom}(g),\ \mathbf{P}(g(G)) \ \vdash\ \mathbf{Q} \qquad \mathbf{H},\ \neg G \in \mathrm{dom}(g),\ \mathbf{P}((\mathrm{dom}(g) \mathbin{\lhd\mkern-14mu-} f)(G)) \ \vdash\ \mathbf{Q}}{\mathbf{H},\ \mathbf{P}((f \mathbin{\lhd\mkern-14mu+} g)(G)) \ \vdash\ \mathbf{Q}}$$	where $\mathbf{P}$ is WD strict	A
*	OV_R	$$\frac{\mathbf{H},\ G \in \mathrm{dom}(g) \ \vdash\ \mathbf{Q}(g(G)) \qquad \mathbf{H},\ \neg G \in \mathrm{dom}(g) \ \vdash\ \mathbf{Q}((\mathrm{dom}(g) \mathbin{\lhd\mkern-14mu-} f)(G))}{\mathbf{H} \ \vdash\ \mathbf{Q}((f \mathbin{\lhd\mkern-14mu+} g)(G))}$$	where $\mathbf{Q}$ is WD strict	A
*	DIS_BINTER_R	$$\frac{\mathbf{H} \ \vdash\ f^{-1} \in A \mathrel{+\mkern-15mu\rightarrow} B \qquad \mathbf{H} \ \vdash\ \mathbf{Q}(f[S] \cap f[T])}{\mathbf{H} \ \vdash\ \mathbf{Q}(f[S \cap T])}$$	where A and B denote types.	M
*	DIS_BINTER_L	$$\frac{\mathbf{H} \ \vdash\ f^{-1} \in A \mathrel{+\mkern-15mu\rightarrow} B \qquad \mathbf{H},\ \mathbf{Q}(f[S] \cap f[T]) \ \vdash\ \mathbf{G}}{\mathbf{H},\ \mathbf{Q}(f[S \cap T]) \ \vdash\ \mathbf{G}}$$	where A and B denote types.	M
*	DIS_SETMINUS_R	$$\frac{\mathbf{H} \ \vdash\ f^{-1} \in A \mathrel{+\mkern-15mu\rightarrow} B \qquad \mathbf{H} \ \vdash\ \mathbf{Q}(f[S] \setminus f[T])}{\mathbf{H} \ \vdash\ \mathbf{Q}(f[S \setminus T])}$$	where A and B denote types.	M
*	DIS_SETMINUS_L	$$\frac{\mathbf{H} \ \vdash\ f^{-1} \in A \mathrel{+\mkern-15mu\rightarrow} B \qquad \mathbf{H},\ \mathbf{Q}(f[S] \setminus f[T]) \ \vdash\ \mathbf{G}}{\mathbf{H},\ \mathbf{Q}(f[S \setminus T]) \ \vdash\ \mathbf{G}}$$	where A and B denote types.	M
*	SIM_REL_IMAGE_R	$$\frac{\mathbf{H} \ \vdash\ WD(\mathbf{Q}(\{f(E)\})) \qquad \mathbf{H} \ \vdash\ \mathbf{Q}(\{f(E)\})}{\mathbf{H} \ \vdash\ \mathbf{Q}(f[\{E\}])}$$		M
*	SIM_REL_IMAGE_L	$$\frac{\mathbf{H} \ \vdash\ WD(\mathbf{Q}(\{f(E)\})) \qquad \mathbf{H},\ \mathbf{Q}(\{f(E)\}) \ \vdash\ \mathbf{G}}{\mathbf{H},\ \mathbf{Q}(f[\{E\}]) \ \vdash\ \mathbf{G}}$$		M
*	SIM_FCOMP_R	$$\frac{\mathbf{H} \ \vdash\ WD(\mathbf{Q}(g(f(x)))) \qquad \mathbf{H} \ \vdash\ \mathbf{Q}(g(f(x)))}{\mathbf{H} \ \vdash\ \mathbf{Q}((f\,;g)(x))}$$		M
*	SIM_FCOMP_L	$$\frac{\mathbf{H} \ \vdash\ WD(\mathbf{Q}(g(f(x)))) \qquad \mathbf{H},\ \mathbf{Q}(g(f(x))) \ \vdash\ \mathbf{G}}{\mathbf{H},\ \mathbf{Q}((f\,;g)(x)) \ \vdash\ \mathbf{G}}$$		M
*	FIN_SUBSETEQ_R	$$\frac{\mathbf{H} \ \vdash\ WD(T) \qquad \mathbf{H} \ \vdash\ S \subseteq T \qquad \mathbf{H} \ \vdash\ \mathrm{finite}\,(T)}{\mathbf{H} \ \vdash\ \mathrm{finite}\,(S)}$$	the user has to write the set corresponding to T in the	M

			editing area of the Proof Control Window	
*	FIN_BINTER_R	$\dfrac{\mathbf{H} \ \vdash \ \text{finite}\,(S) \ \lor \ \ldots \ \lor \ \text{finite}\,(T)}{\mathbf{H} \ \vdash \ \text{finite}\,(S \ \cap \ \ldots \ \cap \ T)}$		M
	FIN_KINTER_R	$\dfrac{\mathbf{H} \ \vdash \ \exists s \cdot s \in S \land \text{finite}\,(s)}{\mathbf{H} \ \vdash \ \text{finite}\,(\text{inter}(S))}$	where s is fresh	M
	FIN_QINTER_R	$\dfrac{\mathbf{H} \ \vdash \ \exists s \cdot P \land \text{finite}\,(E)}{\mathbf{H} \ \vdash \ \text{finite}\,(\bigcap s \cdot P \mid E)}$		M
*	FIN_SETMINUS_R	$\dfrac{\mathbf{H} \ \vdash \ \text{finite}\,(S)}{\mathbf{H} \ \vdash \ \text{finite}\,(S \setminus T)}$		M
	FIN_REL	$\dfrac{}{\mathbf{H},\ r \in S \ op \ T,\ \text{finite}\,(S),\ \text{finite}\,(T) \ \vdash \ \text{finite}\,(r)}$	where op denotes a set of relations (any arrow)	A
*	FIN_REL_R	$\dfrac{\mathbf{H} \ \vdash \ WD(S \leftrightarrow T) \qquad \mathbf{H} \ \vdash \ r \in S \leftrightarrow T \qquad \mathbf{H} \ \vdash \ \text{finite}\,(S) \qquad \mathbf{H} \ \vdash \ \text{finite}\,(T)}{\mathbf{H} \ \vdash \ \text{finite}\,(r)}$	the user has to write the set corresponding to $S \leftrightarrow T$ in the editing area of the Proof Control Window	M
*	FIN_REL_IMG_R	$\dfrac{\mathbf{H} \ \vdash \ \text{finite}\,(r)}{\mathbf{H} \ \vdash \ \text{finite}\,(r[s])}$		M
*	FIN_REL_RAN_R	$\dfrac{\mathbf{H} \ \vdash \ \text{finite}\,(r)}{\mathbf{H} \ \vdash \ \text{finite}\,(\text{ran}(r))}$		M
*	FIN_REL_DOM_R	$\dfrac{\mathbf{H} \ \vdash \ \text{finite}\,(r)}{\mathbf{H} \ \vdash \ \text{finite}\,(\text{dom}(r))}$		M
	FIN_FUN_DOM	$\dfrac{}{\mathbf{H},\ f \in S \ op \ T,\ \text{finite}\,(S) \ \vdash \ \text{finite}\,(f)}$	where op is	A

			one of ⇸, →, ⤔, ↣, ⤀, ↠, ⤖	
	FIN_FUN_RAN	$\frac{}{\mathbf{H},\ f \in S\ op\ T,\ \text{finite}(T)\ \vdash\ \text{finite}(f)}$	where op is one of ⤔, ↣, ⤖	A
*	FIN_FUN1_R	$\frac{\mathbf{H}\ \vdash\ WD(S ⇸ T) \qquad \mathbf{H}\ \vdash\ f\ \in\ S ⇸ T \qquad \mathbf{H}\ \vdash\ \text{finite}(S)}{\mathbf{H}\ \vdash\ \text{finite}(f)}$	the user has to write the set corresponding to $S ⇸ T$ in the editing area of the Proof Control Window	M
*	FIN_FUN2_R	$\frac{\mathbf{H}\ \vdash\ WD(S ⇸ T) \qquad \mathbf{H}\ \vdash\ f^{-1}\ \in\ S ⇸ T \qquad \mathbf{H}\ \vdash\ \text{finite}(S)}{\mathbf{H}\ \vdash\ \text{finite}(f)}$	the user has to write the set corresponding to $S ⇸ T$ in the editing area of the Proof Control Window	M
*	FIN_FUN_IMG_R	$\frac{\mathbf{H}\ \vdash\ WD(S ⇸ T) \qquad \mathbf{H}\ \vdash\ f\ \in\ S ⇸ T \qquad \mathbf{H}\ \vdash\ \text{finite}(s)}{\mathbf{H}\ \vdash\ \text{finite}(f[s])}$	the user has to write the set corresponding to $S ⇸ T$ in the editing area of the Proof Control Window	M
*	FIN_FUN_RAN_R	$\frac{\mathbf{H}\ \vdash\ WD(S ⇸ T) \qquad \mathbf{H}\ \vdash\ f\ \in\ S ⇸ T \qquad \mathbf{H}\ \vdash\ \text{finite}(S)}{\mathbf{H}\ \vdash\ \text{finite}(\text{ran}(f))}$	the user has to write the set corresponding to $S ⇸ T$ in the editing area of the Proof Control Window	M
*	FIN_FUN_DOM_R	$\frac{\mathbf{H}\ \vdash\ WD(S ⇸ T) \qquad \mathbf{H}\ \vdash\ f^{-1}\ \in\ S ⇸ T \qquad \mathbf{H}\ \vdash\ \text{finite}(S)}{\mathbf{H}\ \vdash\ \text{finite}(\text{dom}(f))}$	the user has to write the set corresponding to $S ⇸ T$ in the editing area of the Proof Control Window	M

*	LOWER_BOUND_L	$\dfrac{\mathbf{H} \;\vdash\; \mathrm{finite}(S)}{\mathbf{H} \;\vdash\; \exists n \cdot (\forall x \cdot x \in S \;\Rightarrow\; n \leq x)}$	S must not contain any bound variable	M
*	LOWER_BOUND_R	$\dfrac{\mathbf{H} \;\vdash\; \mathrm{finite}(S)}{\mathbf{H} \;\vdash\; \exists n \cdot (\forall x \cdot x \in S \;\Rightarrow\; x \geq n)}$	S must not contain any bound variable	M
*	UPPER_BOUND_L	$\dfrac{\mathbf{H} \;\vdash\; \mathrm{finite}(S)}{\mathbf{H} \;\vdash\; \exists n \cdot (\forall x \cdot x \in S \;\Rightarrow\; n \geq x)}$	S must not contain any bound variable	M
*	UPPER_BOUND_R	$\dfrac{\mathbf{H} \;\vdash\; \mathrm{finite}(S)}{\mathbf{H} \;\vdash\; \exists n \cdot (\forall x \cdot x \in S \;\Rightarrow\; x \leq n)}$	S must not contain any bound variable	M
*	FIN_LT_0	$\dfrac{\mathbf{H} \;\vdash\; \exists n \cdot (\forall x \cdot x \in S \;\Rightarrow\; n \leq x) \qquad \mathbf{H} \;\vdash\; S \subseteq \mathbb{Z} \setminus \mathbb{N}_1}{\mathbf{H} \;\vdash\; \mathrm{finite}(S)}$		M
*	FIN_GE_0	$\dfrac{\mathbf{H} \;\vdash\; \exists n \cdot (\forall x \cdot x \in S \;\Rightarrow\; x \leq n) \qquad \mathbf{H} \;\vdash\; S \subseteq \mathbb{N}}{\mathbf{H} \;\vdash\; \mathrm{finite}(S)}$		M
	CARD_INTERV	$\dfrac{\mathbf{H},\, a \leq b \;\vdash\; \mathbf{Q}(b-a+1) \qquad \mathbf{H},\, b < a \;\vdash\; \mathbf{Q}(0)}{\mathbf{H} \;\vdash\; \mathbf{Q}(\mathrm{card}\,(a\,..\,b))}$	where $\mathbf{Q}$ is WD strict	M
	CARD_EMPTY_INTERV	$\dfrac{\mathbf{H},\, a \leq b,\, \mathbf{P}(b-a+1) \;\vdash\; \mathbf{Q} \qquad \mathbf{H},\, b < a,\, \mathbf{P}(0) \;\vdash\; \mathbf{Q}}{\mathbf{H},\, \mathbf{P}(\mathrm{card}\,(a\,..\,b)) \;\vdash\; \mathbf{Q}}$	where $\mathbf{P}$ is WD strict	M
*	DERIV_LE_CARD	$\dfrac{\mathbf{H} \;\vdash\; S \subseteq T}{\mathbf{H} \;\vdash\; \mathrm{card}(S) \leq \mathrm{card}(T)}$	S and T bear the same type	M
*	DERIV_GE_CARD	$\dfrac{\mathbf{H} \;\vdash\; T \subseteq S}{\mathbf{H} \;\vdash\; \mathrm{card}(S) \geq \mathrm{card}(T)}$	S and T bear the same type	M
*	DERIV_LT_CARD	$\dfrac{\mathbf{H} \;\vdash\; S \subset T}{\mathbf{H} \;\vdash\; \mathrm{card}(S) < \mathrm{card}(T)}$	S and T bear the same type	M

*	DERIV_GT_CARD	$\dfrac{\mathbf{H} \;\vdash\; T \subset S}{\mathbf{H} \;\vdash\; \mathrm{card}(S) > \mathrm{card}(T)}$	S and T bear the same type	M
*	DERIV_EQUAL_CARD	$\dfrac{\mathbf{H} \;\vdash\; S = T}{\mathbf{H} \;\vdash\; \mathrm{card}(S) = \mathrm{card}(T)}$	S and T bear the same type	M
	SIMP_CARD_SETMINUS_L	$\dfrac{\mathbf{H},\, \mathbf{P}(\mathrm{card}(S \setminus T)) \;\vdash\; \mathrm{finite}(S) \qquad \mathbf{H},\, \mathbf{P}(\mathrm{card}(S) - \mathrm{card}(S \cap T)) \;\vdash\; \mathbf{G}}{\mathbf{H},\, \mathbf{P}(\mathrm{card}(S \setminus T)) \;\vdash\; \mathbf{G}}$		M
	SIMP_CARD_SETMINUS_R	$\dfrac{\mathbf{H} \;\vdash\; \mathrm{finite}(S) \qquad \mathbf{H} \;\vdash\; \mathbf{P}(\mathrm{card}(S) - \mathrm{card}(S \cap T))}{\mathbf{H} \;\vdash\; \mathbf{P}(\mathrm{card}(S \setminus T))}$		M
	SIMP_CARD_CPROD_L	$\dfrac{\mathbf{H},\, \mathbf{P}(\mathrm{card}(S \times T)) \;\vdash\; \mathrm{finite}(S) \qquad \mathbf{H},\, \mathbf{P}(\mathrm{card}(S \times T)) \;\vdash\; \mathrm{finite}(T) \qquad \mathbf{H},\, \mathbf{P}(\mathrm{card}(S) * \mathrm{card}(T)) \;\vdash\; \mathbf{G}}{\mathbf{H},\, \mathbf{P}(\mathrm{card}(S \times T)) \;\vdash\; \mathbf{G}}$		M
	SIMP_CARD_CPROD_R	$\dfrac{\mathbf{H} \;\vdash\; \mathrm{finite}(S) \qquad \mathbf{H} \;\vdash\; \mathrm{finite}(T) \qquad \mathbf{H} \;\vdash\; \mathbf{P}(\mathrm{card}(S) * \mathrm{card}(T))}{\mathbf{H} \;\vdash\; \mathbf{P}(\mathrm{card}(S \times T))}$		M
*	FORALL_INST	$\dfrac{\mathbf{H} \;\vdash\; WD(E) \qquad \mathbf{H}, [x := E]\mathbf{P} \;\vdash\; \mathbf{G}}{\mathbf{H}, \forall x \cdot \mathbf{P} \;\vdash\; \mathbf{G}}$	x is instantiated with E	M
*	FORALL_INST_MP	$\dfrac{\mathbf{H} \;\vdash\; WD(E) \qquad \mathbf{H}, WD(E) \;\vdash\; [x := E]\mathbf{P} \qquad \mathbf{H}, WD(E), [x := E]\mathbf{Q} \;\vdash\; \mathbf{G}}{\mathbf{H}, \forall x \cdot \mathbf{P} \Rightarrow \mathbf{Q} \;\vdash\; \mathbf{G}}$	x is instantiated with E and a Modus Ponens is applied	M
*	FORALL_INST_MT	$\dfrac{\mathbf{H} \;\vdash\; WD(E) \qquad \mathbf{H}, WD(E) \;\vdash\; [x := E]\neg\mathbf{Q} \qquad \mathbf{H}, WD(E), [x := E]\neg\mathbf{P} \;\vdash\; \mathbf{G}}{\mathbf{H}, \forall x \cdot \mathbf{P} \Rightarrow \mathbf{Q} \;\vdash\; \mathbf{G}}$	x is instantiated with E and a Modus Tollens is applied	M
*	CUT	$\dfrac{\mathbf{H} \;\vdash\; WD(\mathbf{P}) \qquad \mathbf{H}, WD(\mathbf{P}) \;\vdash\; \mathbf{P} \qquad \mathbf{H}, WD(\mathbf{P}), \mathbf{P} \;\vdash\; \mathbf{G}}{\mathbf{H} \;\vdash\; \mathbf{G}}$	hypothesis $\mathbf{P}$ is added	M
*	EXISTS_INST	$\dfrac{\mathbf{H} \;\vdash\; WD(E) \qquad \mathbf{H} \;\vdash\; \mathbf{P}(E)}{\mathbf{H} \;\vdash\; \exists x \cdot \mathbf{P}(x)}$	x is instantiated with E	M

*	DISTINCT_CASE	$$\frac{\mathbf{H} \ \vdash \ WD(\mathbf{P}) \qquad \mathbf{H}, WD(\mathbf{P}), \mathbf{P} \ \vdash \ \mathbf{G} \qquad \mathbf{H}, WD(\mathbf{P}), \neg\mathbf{P} \ \vdash \ \mathbf{G}}{\mathbf{H} \ \vdash \ \mathbf{G}}$$	case distinction on predicate $\mathbf{P}$	M
*	ONE_POINT_L	$$\frac{\mathbf{H} \ \vdash \ WD(E) \qquad \mathbf{H}, \forall x, \ldots, \ldots, z \cdot [y := E]\mathbf{P} \land \ldots \land \ldots \land [y := E]\mathbf{Q} \Rightarrow [y := E]\mathbf{R} \ \vdash \ \mathbf{G}}{\mathbf{H}, \forall x, \ldots, y, \ldots, z \cdot \mathbf{P} \land \ldots \land y = E \land \ldots \land \mathbf{Q} \Rightarrow \mathbf{R} \ \vdash \ \mathbf{G}}$$	The rule can be applied with $\forall$ as well as with $\exists$	A
*	ONE_POINT_R	$$\frac{\mathbf{H} \ \vdash \ WD(E) \qquad \mathbf{H} \ \vdash \ \forall x, \ldots, \ldots, z \cdot [y := E]\mathbf{P} \land \ldots \land \ldots \land [y := E]\mathbf{Q} \Rightarrow [y := E]\mathbf{R}}{\mathbf{H} \ \vdash \ \forall x, \ldots, y, \ldots, z \cdot \mathbf{P} \land \ldots \land y = E \land \ldots \land \mathbf{Q} \Rightarrow \mathbf{R}}$$	The rule can be applied with $\forall$ as well as with $\exists$	A
*	SIM_OV_REL	$$\frac{\mathbf{H} \vdash x \in A \qquad \mathbf{H} \vdash y \in B}{\mathbf{H}, f \in A \ op \ B \vdash f \mathbin{<\!\!+} \{x \mapsto y\} \in A \leftrightarrow B}$$	where op is one of $\leftrightarrow$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow}$, $\mathrel{\leftrightarrow\!\!\!\!\rightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, $⇸$, $\rightarrow$, $⤔$, $\rightarrowtail$, $⤀$, $\twoheadrightarrow$, $⤖$	A
*	SIM_OV_TREL	$$\frac{\mathbf{H} \vdash x \in A \qquad \mathbf{H} \vdash y \in B}{\mathbf{H}, f \in A \ op \ B \vdash f \mathbin{<\!\!+} \{x \mapsto y\} \in A \mathrel{\leftarrow\!\!\!\!\leftrightarrow} B}$$	where op is one of $\mathrel{\leftarrow\!\!\!\!\leftrightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, $\rightarrow$, $\rightarrowtail$, $\twoheadrightarrow$, $⤖$	A
*	SIM_OV_PFUN	$$\frac{\mathbf{H} \vdash x \in A \qquad \mathbf{H} \vdash y \in B}{\mathbf{H}, f \in A \ op \ B \vdash f \mathbin{<\!\!+} \{x \mapsto y\} \in A ⇸ B}$$	where op is one of $⇸$, $\rightarrow$, $⤔$, $\rightarrowtail$, $⤀$, $\twoheadrightarrow$, $⤖$	A
*	SIM_OV_TFUN	$$\frac{\mathbf{H} \vdash x \in A \qquad \mathbf{H} \vdash y \in B}{\mathbf{H}, f \in A \ op \ B \vdash f \mathbin{<\!\!+} \{x \mapsto y\} \in A \rightarrow B}$$	where op is one of $\rightarrow$, $\rightarrowtail$, $\twoheadrightarrow$, $⤖$	A
	INDUC_NAT	$$\frac{\mathbf{H} \ \vdash \ x \in \mathbb{N} \qquad \mathbf{H}, x = 0 \ \vdash \ \mathbf{P}(x) \qquad \mathbf{H}, n \in \mathbb{N}, \mathbf{P}(n) \ \vdash \ \mathbf{P}(n+1)}{\mathbf{H} \ \vdash \ \mathbf{P}(x)}$$	x of type $\mathbb{Z}$ appears free in $\mathbf{P}$; n is introduced as a fresh identifier	M
	INDUC_NAT_COMPL	$$\frac{\mathbf{H} \ \vdash \ x \in \mathbb{N} \qquad \mathbf{H} \ \vdash \ \mathbf{P}(0) \qquad \mathbf{H}, n \in \mathbb{N}, \forall k \cdot 0 \leq k \land k < n \Rightarrow \mathbf{P}(k) \ \vdash \ \mathbf{P}(n)}{\mathbf{H} \ \vdash \ \mathbf{P}(x)}$$	x of type $\mathbb{Z}$ appears free in $\mathbf{P}$; n is introduced as a fresh identifier	M

Those following rules have been implemented in the reasoner GeneralizedModusPonens.

	Name	Rule	Side Condition	A/M
*	GENMP_HYP_HYP	$\dfrac{P, \varphi(\top) \vdash G}{P, \varphi(P^{\dagger}) \vdash G}$	see below for $P^{\dagger}$	A
*	GENMP_NOT_HYP_HYP	$\dfrac{nP^{\dagger}, \varphi(\bot) \vdash G}{nP^{\dagger}, \varphi(P) \vdash G}$	see below for $P^{\dagger}$	A
*	GENMP_HYP_GOAL	$\dfrac{P \vdash \varphi(\top)}{P \vdash \varphi(P^{\dagger})}$	see below for $P^{\dagger}$	A
*	GENMP_NOT_HYP_GOAL	$\dfrac{nP^{\dagger} \vdash \varphi(\bot)}{nP^{\dagger} \vdash \varphi(P)}$	see below for $P^{\dagger}$	A
*	GENMP_GOAL_HYP	$\dfrac{H, \varphi(\bot) \vdash \neg nG^{\dagger}}{H, \varphi(G) \vdash \neg nG^{\dagger}}$	see below for $nG^{\dagger}$	A
*	GENMP_NOT_GOAL_HYP	$\dfrac{H, \varphi(\top) \vdash \neg G}{H, \varphi(G^{\dagger}) \vdash \neg G}$	see below for $G^{\dagger}$	A
*	GENMP_OR_GOAL_HYP	$\dfrac{H, \varphi(\bot) \vdash G_1 \lor \cdots \lor \neg nG_i^{\dagger} \lor \cdots \lor G_n}{H, \varphi(G_i) \vdash G_1 \lor \cdots \lor \neg nG_i^{\dagger} \lor \cdots \lor G_n}$	see below for $nG_i^{\dagger}$	A
*	GENMP_OR_NOT_GOAL_HYP	$\dfrac{H, \varphi(\top) \vdash G_1 \lor \cdots \lor \neg G_i \lor \cdots \lor G_n}{H, \varphi(G_i^{\dagger}) \vdash G_1 \lor \cdots \lor \neg G_i \lor \cdots \lor G_n}$	see below for $G_i^{\dagger}$	A

Thos following rules have been implemented in the MembershipGoal reasoner.

	Name	Rule	Side Condition	A/M
*	SUBSET_SUBSETEQ	$A \subset B \vdash A \subseteq B$		A
*	DOM_SUBSET	$A \subseteq B \vdash \mathrm{dom}(A) \subseteq \mathrm{dom}(B)$		A
*	RAN_SUBSET	$A \subseteq B \vdash \mathrm{ran}(A) \subseteq \mathrm{ran}(B)$		A
*	EQUAL_SUBSETEQ_LR	$A = B \vdash A \subseteq B$		A
*	EQUAL_SUBSETEQ_RL	$A = B \vdash B \subseteq A$		A
*	IN_DOM_CPROD	$x \in \mathrm{dom}(A \times B) \vdash x \in A$		A
*	IN_RAN_CPROD	$y \in \mathrm{ran}(A \times B) \vdash y \in B$		A

*	`IN_DOM_REL`	$x \mapsto y \in f \vdash x \in \mathrm{dom}(f)$		A
*	`IN_RAN_REL`	$x \mapsto y \in f \vdash y \in \mathrm{ran}(f)$		A
*	`SETENUM_SUBSET`	$\{a, \cdots, x, \cdots, z\} \subseteq A \vdash x \in A$		A
*	`OVR_RIGHT_SUBSET`	$f \mathbin{\lhd\mkern-14mu-} \cdots \mathbin{\lhd\mkern-14mu-} g \mathbin{\lhd\mkern-14mu-} \cdots \mathbin{\lhd\mkern-14mu-} h \subseteq A \vdash g \mathbin{\lhd\mkern-14mu-} \cdots \mathbin{\lhd\mkern-14mu-} h \subseteq A$		A
*	`RELSET_SUBSET_CPROD`	$f \in A \; op \; B \vdash f \subseteq A \times B$	where op is one of $\leftrightarrow$, $\leftarrow\mkern-14mu\leftrightarrow$, $\leftrightarrow\mkern-14mu\rightarrow$, $\leftarrow\mkern-14mu\leftrightarrow\mkern-14mu\rightarrow$, $\mapsto\!\!\!\!\!+$, $\rightarrow$, $\rightarrowtail\!\!\!\!\!+$, $\rightarrowtail$, $\twoheadrightarrow\!\!\!\!\!+$, $\twoheadrightarrow$, $\twoheadrightarrow\!\!\!\!\!\!\rightarrowtail$	A
*	`DERIV_IN_SUBSET`	$x \in A,\;\; A \subseteq B \vdash x \in B$		A

The conventions used in this table are described in Variations in HYP, CNTR and GenMP.

P	**P**†	**nP**†	**Side Condition**
$a = b$	$a = b, \quad b = a$ $a \leq b, \quad b \geq a$ $a \geq b, \quad b \leq a$	$\neg a = b, \quad \neg b = a$ $a > b, \quad b < a$ $a < b, \quad b > a$	where a and b are integers
$a < b$	$a < b, \quad b > a$ $a \leq b, \quad b \geq a$ $\neg a = b, \quad \neg b = a$	$a \geq b, \quad b \leq a$ $a > b, \quad b < a$ $a = b, \quad b = a$	
$a > b$	$a > b, \quad b < a$ $a \geq b, \quad b \leq a$ $\neg a = b, \quad \neg b = a$	$a \leq b, \quad b \geq a$ $a < b, \quad b > a$ $a = b, \quad b = a$	
$a \leq b$	$a \leq b, \quad b \geq a$	$a > b, \quad b < a$	
$a \geq b$	$a \geq b, \quad b \leq a$	$a < b, \quad b > a$	
$\neg a = b$	$\neg a = b, \quad \neg b = a$	$a = b, \quad b = a$	
$A = B$	$A = B, \quad B = A$ $A \subseteq B, \quad B \subseteq A$ $\neg A \subset B, \quad \neg B \subset A$	$\neg A = B, \quad \neg B = A$ $\neg A \subseteq B, \quad \neg B \subseteq A$ $A \subset B, \quad B \subset A$	where A and B are sets
$A \subseteq B$	$A \subseteq B, \neg B \subset A$	$\neg A \subseteq B, B \subset A$	
$A \subset B$	$A \subset B, \quad A \subseteq B$ $\neg B \subset A, \quad \neg B \subseteq A$ $\neg A = B, \quad \neg B = A$	$\neg A \subset B, \quad \neg A \subseteq B$ $B \subset A, \quad B \subseteq A$ $A = B, \quad B = A$	
$\neg A = B$	$\neg A = B, \quad \neg B = A$	$A = B, \quad B = A$	
$\neg A \subseteq B$	$\neg A \subseteq B, \quad \neg A \subset B$ $\neg A = B, \quad \neg B = A$	$A \subseteq B, \quad A \subset B$ $A = B, \quad B = A$	
$\neg A \subset B$	$\neg A \subset B$	$A \subset B$	

$e = f$	$e = f, \quad f = e$	$\neg e = f, \quad \neg f = e$	where e and f are scalars
$\neg e = f$	$\neg e = f, \quad \neg f = e$	$e = f, \quad f = e$	
$\mathbf{P}$	$\mathbf{P}$	$\neg\mathbf{P}$	
$\neg\mathbf{P}$		$\mathbf{P}$	

See also Extension Proof Rules#Inference Rules.